8. $y = c_1 + c_2 e^t + c_3 e^{-t} - \ln \sin t + \ln(\cos t + 1) + \frac{1}{2} e^t \int_{t_0}^{t} \left(e^{-s} / \sin s \right) ds$
$+ \frac{1}{2} e^{-t} \int_{t_0}^{t} \left(e^{s} / \sin s \right) ds$
9. $c_1 = 0,\ c_2 = 2,\ c_3 = 1$ in answer to Problem 4
10. $c_1 = 2,\ c_2 = \frac{7}{8},\ c_3 = -\frac{7}{8},\ c_4 = \frac{1}{2}$ in answer to Problem 6

See SSM for detailed solutions to 11, 14

11. $c_1 = \frac{3}{2},\ c_2 = \frac{1}{2},\ c_3 = -\frac{5}{2},\ t_0 = 0$ in answer to Problem 7
12. $c_1 = 3,\ c_2 = 0,\ c_3 = -e^{\pi/2},\ t_0 = \pi/2$ in answer to Problem 8
13. $Y(x) = x^4/15$
14. $Y(t) = \frac{1}{2} \int_{t_0}^{t} [e^{t-s} - \sin(t-s) - \cos(t-s)] g(s)\, ds$
15. $Y(t) = \frac{1}{2} \int_{t_0}^{t} [\sinh(t-s) - \sin(t-s)] g(s)\, ds$

16

16. $Y(t) = \frac{1}{2} \int_{t_0}^{t} e^{(t-s)} (t-s)^2 g(s)\, ds; \quad Y(t) = -te^t \ln |t|$
17. $Y(x) = \frac{1}{2} \int_{x_0}^{x} [(x/t^2) - 2(x^2/t^3) + (x^3/t^4)] g(t)\, dt$

CHAPTER 5

Section 5.1, page 237

2, 5, 9, 12, 13

1. $\rho = 1$
2. $\rho = 2$
3. $\rho = \infty$
4. $\rho = \frac{1}{2}$
5. $\rho = \frac{1}{2}$
6. $\rho = 1$
7. $\rho = 3$
8. $\rho = e$
9. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \rho = \infty$
10. $\sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \rho = \infty$
11. $1 + (x-1), \quad \rho = \infty$
12. $1 - 2(x+1) + (x+1)^2, \quad \rho = \infty$
13. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}, \quad \rho = 1$
14. $\sum_{n=0}^{\infty} (-1)^n x^n, \quad \rho = 1$
15. $\sum_{n=0}^{\infty} x^n, \quad \rho = 1$
16. $\sum_{n=0}^{\infty} (-1)^{n+1} (x-2)^n, \quad \rho = 1$
17. $y' = 1 + 2^2 x + 3^2 x^2 + 4^2 x^3 + \cdots + (n+1)^2 x^n + \cdots$
$y'' = 2^2 + 3^2 \cdot 2x + 4^2 \cdot 3x^2 + 5^2 \cdot 4x^3 + \cdots + (n+2)^2 (n+1) x^n + \cdots$

18, 19, 23, 25, 28

18. $y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots + (n+1) a_{n+1} x^n + \cdots$
$= \sum_{n=1}^{\infty} n a_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n$
$y'' = 2a_2 + 6a_3 x + 12 a_4 x^2 + 20 a_5 x^3 + \cdots + (n+2)(n+1) a_{n+2} x^n + \cdots$
$= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$
21. $\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$
22. $\sum_{n=2}^{\infty} a_{n-2} x^n$
23. $\sum_{n=0}^{\infty} (n+1) a_n x^n$
24. $\sum_{n=0}^{\infty} [(n+2)(n+1) a_{n+2} - n(n-1) a_n] x^n$
25. $\sum_{n=0}^{\infty} [(n+2)(n+1) a_{n+2} + n a_n] x^n$
26. $a_1 + \sum_{n=1}^{\infty} [(n+1) a_{n+1} + a_{n-1}] x^n$
27. $\sum_{n=0}^{\infty} [(n+1) n a_{n+1} + a_n] x^n$
28. $a_n = (-2)^n a_0 / n!, \quad n = 1, 2, \ldots; \quad a_0 e^{-2x}$

Section 5.2, page 247

1. $a_{n+2} = a_n/(n+2)(n+1)$
$$y_1(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = \cosh x$$
$$y_2(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = \sinh x$$

See SSM for detailed solutions to 2

2. $a_{n+2} = a_n/(n+2)$
$$y_1(x) = 1 + \frac{x^2}{2} + \frac{x^4}{2\cdot 4} + \frac{x^6}{2\cdot 4\cdot 6} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$$
$$y_2(x) = x + \frac{x^3}{3} + \frac{x^5}{3\cdot 5} + \frac{x^7}{3\cdot 5\cdot 7} + \cdots = \sum_{n=0}^{\infty} \frac{2^n n! x^{2n+1}}{(2n+1)!}$$

3

3. $(n+2)a_{n+2} - a_{n+1} - a_n = 0$
$y_1(x) = 1 + \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 + \frac{1}{6}(x-1)^4 + \cdots$
$y_2(x) = (x-1) + \frac{1}{2}(x-1)^2 + \frac{1}{2}(x-1)^3 + \frac{1}{4}(x-1)^4 + \cdots$

4. $a_{n+4} = -k^2 a_n/(n+4)(n+3); \quad a_2 = a_3 = 0$
$$y_1(x) = 1 - \frac{k^2x^4}{3\cdot 4} + \frac{k^4x^8}{3\cdot 4\cdot 7\cdot 8} - \frac{k^6x^{12}}{3\cdot 4\cdot 7\cdot 8\cdot 11\cdot 12} + \cdots$$
$$= 1 + \sum_{m=0}^{\infty} \frac{(-1)^{m+1}(k^2x^4)^{m+1}}{3\cdot 4\cdot 7\cdot 8\cdots(4m+3)(4m+4)}$$
$$y_2(x) = x - \frac{k^2x^5}{4\cdot 5} + \frac{k^4x^9}{4\cdot 5\cdot 8\cdot 9} - \frac{k^6x^{13}}{4\cdot 5\cdot 8\cdot 9\cdot 12\cdot 13} + \cdots$$
$$= x\left[1 + \sum_{m=0}^{\infty} \frac{(-1)^{m+1}(k^2x^4)^{m+1}}{4\cdot 5\cdot 8\cdot 9\cdots(4m+4)(4m+5)}\right]$$
Hint: Let $n = 4m$ in the recurrence relation, $m = 1, 2, 3, \ldots$.

5, 8

5. $(n+2)(n+1)a_{n+2} - n(n+1)a_{n+1} + a_n = 0, \quad n \geq 1; \quad a_2 = -\frac{1}{2}a_0$
$y_1(x) = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{24}x^4 + \cdots, \quad y_2(x) = x - \frac{1}{6}x^3 - \frac{1}{12}x^4 - \frac{1}{24}x^5 + \cdots$

6. $a_{n+2} = -(n^2 - 2n + 4)a_n/[2(n+1)(n+2)], \quad n \geq 2; \quad a_2 = -a_0, \quad a_3 = -\frac{1}{4}a_1$
$y_1(x) = 1 - x^2 + \frac{1}{6}x^4 - \frac{1}{30}x^6 + \cdots, \quad y_2(x) = x - \frac{1}{4}x^3 + \frac{7}{160}x^5 - \frac{19}{1920}x^7 + \cdots$

7. $a_{n+2} = -a_n/(n+1), \quad n = 0, 1, 2, \ldots$
$$y_1(x) = 1 - \frac{x^2}{1} + \frac{x^4}{1\cdot 3} - \frac{x^6}{1\cdot 3\cdot 5} + \cdots = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{1\cdot 3\cdot 5\cdots(2n-1)}$$
$$y_2(x) = x - \frac{x^3}{2} + \frac{x^5}{2\cdot 4} - \frac{x^7}{2\cdot 4\cdot 6} + \cdots = x + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2\cdot 4\cdot 6\cdots(2n)}$$

8. $a_{n+2} = -[(n+1)^2 a_{n+1} + a_n + a_{n-1}]/(n+1)(n+2), \quad n = 1, 2, \ldots$
$a_2 = -(a_0 + a_1)/2$
$y_1(x) = 1 - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{12}(x-1)^4 + \cdots$
$y_2(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{6}(x-1)^3 - \frac{1}{6}(x-1)^4 + \cdots$

9. $(n+2)(n+1)a_{n+2} + (n-2)(n-3)a_n = 0; \quad n = 0, 1, 2, \ldots$
$y_1(x) = 1 - 3x^2, \quad y_2(x) = x - x^3/3$

10. $4(n+2)a_{n+2} - (n-2)a_n = 0; \quad n = 0, 1, 2, \ldots$
$$y_1(x) = 1 - \frac{x^2}{4}, \quad y_2(x) = x - \frac{x^3}{12} - \frac{x^5}{240} - \frac{x^7}{2240} - \cdots - \frac{x^{2n+1}}{4^n(2n-1)(2n+1)} - \cdots$$

11. $3(n+2)a_{n+2} - (n+1)a_n = 0; \quad n = 0, 1, 2, \ldots$

$y_1(x) = 1 + \frac{x^2}{6} + \frac{x^4}{24} + \frac{5}{432}x^6 + \cdots + \frac{3 \cdot 5 \cdots (2n-1)}{3^n \cdot 2 \cdot 4 \cdots (2n)}x^{2n} + \cdots$

$y_2(x) = x + \frac{2}{9}x^3 + \frac{8}{135}x^5 + \frac{16}{945}x^7 + \cdots + \frac{2 \cdot 4 \cdots (2n)}{3^n \cdot 3 \cdot 5 \cdots (2n+1)}x^{2n+1} + \cdots$

12. $(n+2)(n+1)a_{n+2} - (n+1)na_{n+1} + (n-1)a_n = 0; \quad n = 0, 1, 2, \ldots$

$y_1(x) = 1 + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots + \frac{x^n}{n!} + \cdots, \quad y_2(x) = x$

13. $2(n+2)(n+1)a_{n+2} + (n+3)a_n = 0; \quad n = 0, 1, 2, \ldots$

$y_1(x) = 1 - \frac{3}{4}x^2 + \frac{5}{32}x^4 - \frac{7}{384}x^6 + \cdots + (-1)^n\frac{3 \cdot 5 \cdots (2n+1)}{2^n(2n)!}x^{2n} + \cdots$

$y_2(x) = x - \frac{x^3}{3} + \frac{x^5}{20} - \frac{x^7}{210} + \cdots + (-1)^n\frac{4 \cdot 6 \cdots (2n+2)}{2^n(2n+1)!}x^{2n+1} + \cdots$

See SSM for detailed solutions to 14, 16a, 19

14. $2(n+2)(n+1)a_{n+2} + 3(n+1)a_{n+1} + (n+3)a_n = 0; \quad n = 0, 1, 2, \ldots$

$y_1(x) = 1 - \frac{3}{4}(x-2)^2 + \frac{3}{8}(x-2)^3 + \frac{1}{64}(x-2)^4 + \cdots$

$y_2(x) = (x-2) - \frac{3}{4}(x-2)^2 + \frac{1}{24}(x-2)^3 + \frac{9}{64}(x-2)^4 + \cdots$

15. (a) $y = 2 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots$ (c) about $|x| < 0.7$

16. (a) $y = -1 + 3x + x^2 - \frac{3}{4}x^3 - \frac{1}{6}x^4 + \cdots$ (c) about $|x| < 0.7$

17. (a) $y = 4 - x - 4x^2 + \frac{1}{2}x^3 + \frac{4}{3}x^4 + \cdots$ (c) about $|x| < 0.5$

18. (a) $y = -3 + 2x - \frac{3}{2}x^2 - \frac{1}{2}x^3 - \frac{1}{8}x^4 + \cdots$ (c) about $|x| < 0.9$

19. $y_1(x) = 1 - \frac{1}{3}(x-1)^3 - \frac{1}{12}(x-1)^4 + \frac{1}{18}(x-1)^6 + \cdots$

$y_2(x) = (x-1) - \frac{1}{4}(x-1)^4 - \frac{1}{20}(x-1)^5 + \frac{1}{28}(x-1)^7 + \cdots$

21. (a) $y_1(x) = 1 - \frac{\lambda}{2!}x^2 + \frac{\lambda(\lambda-4)}{4!}x^4 - \frac{\lambda(\lambda-4)(\lambda-8)}{6!}x^6 + \cdots$

$y_2(x) = x - \frac{\lambda-2}{3!}x^3 + \frac{(\lambda-2)(\lambda-6)}{5!}x^5 - \frac{(\lambda-2)(\lambda-6)(\lambda-10)}{7!}x^7 + \cdots$

(b) $1, \ x, \ 1 - 2x^2, \ x - \frac{2}{3}x^3, \ 1 - 4x^2 + \frac{4}{3}x^4, \ x - \frac{4}{3}x^3 + \frac{4}{15}x^5$

(c) $1, \ 2x, \ 4x^2 - 2, \ 8x^3 - 12x, \ 16x^4 - 48x^2 + 12, \ 32x^5 - 160x^3 + 120x$

22b, 23, 26

22. (b) $y = x - x^3/6 + \cdots$

Section 5.3, page 253

1, 6, 9afh

1. $\phi''(0) = -1, \quad \phi'''(0) = 0, \quad \phi^{\text{iv}}(0) = 3$
2. $\phi''(0) = 0, \quad \phi'''(0) = -2, \quad \phi^{\text{iv}}(0) = 0$
3. $\phi''(1) = 0, \quad \phi'''(1) = -6, \quad \phi^{\text{iv}}(1) = 42$
4. $\phi''(0) = 0, \quad \phi'''(0) = -a_0, \quad \phi^{\text{iv}}(0) = -4a_1$
5. $\rho = \infty, \quad \rho = \infty$
6. $\rho = 1, \quad \rho = 3, \quad \rho = 1$
7. $\rho = 1, \quad \rho = \sqrt{3}$
8. $\rho = 1$
9. (a) $\rho = \infty$ (b) $\rho = \infty$ (c) $\rho = \infty$ (d) $\rho = \infty$ (e) $\rho = 1$
(f) $\rho = \sqrt{2}$ (g) $\rho = \infty$ (h) $\rho = 1$ (i) $\rho = 1$ (j) $\rho = 2$
(k) $\rho = \sqrt{3}$ (l) $\rho = 1$ (m) $\rho = \infty$ (n) $\rho = \infty$

See SSM for detailed solutions to 10a

10. (a) $y_1(x) = 1 - \frac{\alpha^2}{2!}x^2 - \frac{(2^2-\alpha^2)\alpha^2}{4!}x^4 - \frac{(4^2-\alpha^2)(2^2-\alpha^2)\alpha^2}{6!}x^6 - \cdots$
$- \frac{[(2m-2)^2-\alpha^2]\cdots(2^2-\alpha^2)\alpha^2}{(2m)!}x^{2m} - \cdots$

$y_2(x) = x + \frac{1-\alpha^2}{3!}x^3 + \frac{(3^2-\alpha^2)(1-\alpha^2)}{5!}x^5 + \cdots$
$+ \frac{[(2m-1)^2-\alpha^2]\cdots(1-\alpha^2)}{(2m+1)!}x^{2m+1} + \cdots$

10b, 11

(b) $y_1(x)$ or $y_2(x)$ terminates with x^n as $\alpha = n$ is even or odd.

(c) $n = 0,\ y = 1;\quad n = 1,\ y = x;\quad n = 2,\ y = 1 - 2x^2;\quad n = 3,\ y = x - \frac{4}{3}x^3$

11. $y_1(x) = 1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{180}x^6 + \cdots,\quad y_2(x) = x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \frac{1}{504}x^7 + \cdots,$
$\rho = \infty$

12. $y_1(x) = 1 - \frac{1}{6}x^3 + \frac{1}{12}x^4 - \frac{1}{40}x^5 + \cdots,\quad y_2(x) = x - \frac{1}{12}x^4 + \frac{1}{20}x^5 - \frac{1}{60}x^6 + \cdots,$
$\rho = \infty$

13. $y_1(x) = 1 + x^2 + \frac{1}{12}x^4 + \frac{1}{120}x^6 + \cdots,\quad y_2(x) = x + \frac{1}{6}x^3 + \frac{1}{60}x^5 + \frac{1}{560}x^7 + \cdots,$
$\rho = \pi/2$

14. $y_1(x) = 1 + \frac{1}{6}x^3 + \frac{1}{12}x^4 - \frac{1}{120}x^6 + \cdots,\quad y_2(x) = x - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{7}{120}x^5 + \cdots,$
$\rho = 1$

15. Cannot specify arbitrary initial conditions at $x = 0$; hence $x = 0$ is a singular point.

16. $y = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = e^x$

17. $y = 1 + \frac{x^2}{2} + \frac{x^4}{2\cdot 4} + \frac{x^6}{2\cdot 4\cdot 6} + \cdots + \frac{x^{2n}}{2^n\cdot n!} + \cdots$

18

18. $y = 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots$

19. $y = 1 + x + x^2 + \cdots + x^n + \cdots = \frac{1}{1-x}$

20. $y = a_0\left(1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots\right) + 2\left(\frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots\right)$

$= a_0e^x + 2\left(e^x - 1 - x - \frac{x^2}{2}\right) = ce^x - 2 - 2x - x^2$

20, 22

21. $y = a_0\left(1 - \frac{x^2}{2} + \frac{x^4}{2^2 2!} - \frac{x^6}{2^3 3!} + \cdots + \frac{(-1)^n x^{2n}}{2^n n!} + \cdots\right)$

$+ \left(x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{2\cdot 4} + \frac{x^5}{3\cdot 5} + \cdots\right)$

$= a_0e^{-x^2/2} + \left(x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{2\cdot 4} + \frac{x^5}{3\cdot 5} + \cdots\right)$

26, 28

23. $1,\ 1 - 3x^2,\ 1 - 10x^2 + \frac{35}{3}x^4;\quad x,\ x - \frac{5}{3}x^3,\ x - \frac{14}{3}x^3 + \frac{21}{5}x^5$

24. (a) $1,\ x,\ (3x^2-1)/2,\ (5x^3-3x)/2,\ (35x^4-30x^2+3)/8,\ (63x^5-70x^3+15x)/8$
(c) $P_1,\ 0;\quad P_2,\ \pm 0.57735;\quad P_3,\ 0,\ \pm 0.77460;\quad P_4,\ \pm 0.33998,\ \pm 0.86114;$
$P_5,\ 0,\ \pm 0.53847, \pm 0.90618$

Section 5.4, page 259

See SSM for detailed solutions to 1, 5

1. $x = 0$, regular
2. $x = 0$, regular; $x = 1$, irregular
3. $x = 0$, irregular; $x = 1$, regular
4. $x = 0$, irregular; $x = \pm 1$, regular
5. $x = 1$, regular; $x = -1$, irregular
6. $x = 0$, regular
7. $x = -3$, regular
8. $x = 0, -1$, regular; $x = 1$, irregular
9. $x = 1$, regular; $x = -2$, irregular
10. $x = 0, 3$, regular
11. $x = 1, -2$, regular
12. $x = 0$, regular
13. $x = 0$, irregular
14. $x = 0$, regular
15. $x = 0$, regular
16. $x = 0, \pm n\pi$, regular
17. $x = 0, \pm n\pi$, regular
18. $x = 0$, irregular; $x = \pm n\pi$, regular
19. $y = a_0\left(1 - \dfrac{x^2}{2\cdot 5} + \dfrac{x^4}{2\cdot 4\cdot 5\cdot 9} - \cdots\right)$
22. Irregular singular point
23. Regular singular point
24. Regular singular point
25. Irregular singular point
26. Irregular singular point
27. Irregular singular point

12, 17

19, 21, 23

25

Section 5.5, page 265

2, 4, 9, 13, 16

1. $y = c_1x^{-1} + c_2x^{-2}$
2. $y = c_1|x+1|^{-1/2} + c_2|x+1|^{-3/2}$
3. $y = c_1x^2 + c_2x^2 \ln|x|$
4. $y = c_1x^{-1}\cos(2\ln|x|) + c_2x^{-1}\sin(2\ln|x|)$
5. $y = c_1x + c_2x\ln|x|$
6. $y = c_1(x-1)^{-3} + c_2(x-1)^{-4}$
7. $y = c_1|x|^{(-5+\sqrt{29})/2} + c_2|x|^{(-5-\sqrt{29})/2}$
8. $y = c_1|x|^{3/2}\cos(\frac{1}{2}\sqrt{3}\ln|x|) + c_2|x|^{3/2}\sin(\frac{1}{2}\sqrt{3}\ln|x|)$
9. $y = c_1x^3 + c_2x^3\ln|x|$
10. $y = c_1(x-2)^{-2}\cos(2\ln|x-2|) + c_2(x-2)^{-2}\sin(2\ln|x-2|)$
11. $y = c_1|x|^{-1/2}\cos(\frac{1}{2}\sqrt{15}\ln|x|) + c_2|x|^{-1/2}\sin(\frac{1}{2}\sqrt{15}\ln|x|)$
12. $y = c_1x + c_2x^4$
13. $y = 2x^{3/2} - x^{-1}$
14. $y = 2x^{-1/2}\cos(2\ln x) - x^{-1/2}\sin(2\ln x)$
15. $y = 2x^2 - 7x^2\ln|x|$
16. $y = x^{-1}\cos(2\ln x)$
17. $\alpha < 1$
18. $\beta > 0$
19. $\gamma = 2$
20. $\alpha > 1$
21. (a) $\alpha < 1$ and $\beta > 0$
    (b) $\alpha < 1$ and $\beta \geq 0$, or $\alpha = 1$ and $\beta > 0$
    (c) $\alpha > 1$ and $\beta > 0$
    (d) $\alpha > 1$ and $\beta \geq 0$, or $\alpha = 1$ and $\beta > 0$
    (e) $\alpha = 1$ and $\beta > 0$
24. $y = c_1x^{-1} + c_2x^2$
25. $y = c_1x^2 + c_2x^2\ln x + \frac{1}{4}\ln x + \frac{1}{4}$
26. $y = c_1x^{-1} + c_2x^{-5} + \frac{1}{12}x$
27. $y = c_1x + c_2x^2 + 3x^2\ln x + \ln x + \frac{3}{2}$
28. $y = c_1\cos(2\ln x) + c_2\sin(2\ln x) + \frac{1}{3}\sin(\ln x)$
29. $y = c_1x^{-3/2}\cos(\frac{3}{2}\ln x) + c_2x^{-3/2}\sin(\frac{3}{2}\ln x)$
31. $x > 0:\quad c_1 = k_1,\ c_2 = k_2;\quad x < 0:\quad c_1(-1)^{r_1} = k_1,\ c_2 = k_2$

17, 21, 21a, 22, 25, 31