

1) The snow plow problem (R.P. Agnew):

It began to snow early in the morning and the snow continued to fall throughout the day at a constant rate. Assume that the speed at which a snow plow is able to clear a road is inversely proportional to the height of the snow. The snow plow started at 5 a.m. and had cleared four miles by 8 a.m. By 12 noon it had cleared another two miles. At what time did it start snowing?

2) Four caterpillars (P.M. Golbart):

Four caterpillars, initially at rest at the four corners of a square centered at the origin, start walking clockwise, each caterpillar walking directly towards the one in front of him. If each caterpillar walks with unit velocity, show that the trajectories satisfy the differential equation $y' = (y - x)/(y + x)$. Solve the equation.

3) Farmer and the pig (P.M. Golbart):

At $t = 0$, a pig, initially at the origin, runs along the x -axis with constant speed v . At $t = 0$, a farmer, initially 20m North from the origin, also runs with constant speed v . If the farmer's instantaneous velocity is always directed towards the instantaneous position of the pig, show that the farmer never gets closer than 10m from the pig.

4) Torricelli's law

A large open cistern has the shape of a hemisphere with radius R and a small circular hole of radius r in its bottom. By Torricelli's law, water flows out of the hole with the velocity $v = \sqrt{2gh}$, where h is the height of the water layer above the hole. If the cistern is filled with water, how long will it take for all water to drain from the cistern?

5) (Boyce & DiPrima) A pond initially contains $V \text{ m}^3$ of water with an undesired chemical in concentration $c_0 \text{ gram/m}^3$. Water containing $c_1 \text{ gram/m}^3$ of this chemical flows into the pond at a rate of $Q \text{ m}^3/\text{min}$. The mixture flows out at the same rate so the amount of water in the pond remains constant. Assume that the chemical is uniformly distributed throughout the pond. Find how the concentration of this chemical in the pond changes with time, $c=c(t)$.

6) (Boyce & DiPrima) A pond forms as water collects in a conical depression of radius a and depth h . Suppose that water flows in at a constant rate k and is lost through evaporation at a rate proportional to the surface area: amount of water evaporated per unit of time is bS , where S is the surface area, b - coefficient of evaporation. Write a differential equation describing how the volume of water in the pond changes with time. Find the equilibrium depth of water in the pond. Is the equilibrium stable? Find a condition that must be satisfied if the pond is not to overflow.