Introduction to Numerical Analysis, Spring 2005

Assignment 2

1. A ball of radius R, made of material with the density ρ_s, is placed into a liquid of density ρ_l, $0 < \rho_s < \rho_l$. We want to determine the depth h to which the ball is submerged.

   Derive an equation $F(x, r) = 0$ determining $x = h/R$ for a given density ratio $r = \rho_s/\rho_l$. Write an m-function `x=depth(r,tol)` that uses Newton method to find x satisfying this equation with a given tolerance, $|F(x, r)| \leq tol$, for a given density ratio r. Explain your choice of the initial approximation x_0. Plot and attach to your report a graph $x = x(r)$ for $0 \leq r \leq 1$.

2. Suppose your computer forgot how to calculate logarithms, so you need to write your own function `y=my_log(x,abs_err)` in Matlab to compute natural logarithms $y = \log(x)$ for $x > 0$ with the absolute error not exceeding `abs_err`.

   A possible algorithm is based on the Taylor expansion of logarithm,

   $$\log(1 - z) = -z - \frac{z^2}{2} - \frac{z^3}{3} - ...$$

   The series converges only if $|z| < 1$ and the convergence is very slow for z close to ± 1. We can circumvent these limitations as follows. Let the value of e be known. If $x > 1$ we find the minimal integer m such that $x \leq e^m$ and use the equality $\log(x) = m + \log(xe^{-m})$. Then $xe^{-m} = 1 - z$, where $0 < z < 1 - 1/e \approx 0.63$, so the series converges quickly (you need to estimate the absolute error if only n first terms have been summed up). For $0 < x < 1$ we have $\log(x) = -\log(1/x)$, so the same algorithm can be used.

   You may use this algorithm or find a better way to compute logarithms (but do not use e^x or any other function which is as difficult to calculate as the logarithmic one).

3. Write an m-function `secant` that solves nonlinear equations by the secant method. Use this program and your function `my_log` to solve the equation $\log(x) = 10$ with the absolute error Δx not exceeding 10^{-3}. Note that there are two sources of error: the log values you get are not exact and the equation is solved numerically. In your written report explain how did you choose the values of the parameters `abs_err` in `my_log` and `tol` in `secant` (see program specification) to achieve the needed accuracy.

4. Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance one can determine the temperature.

   Steinhart-Hart equation for the 10K3A Betatherm thermistor is (A. Kaw)

   $$\frac{1}{T} = a_0 + a_1 \log(R) + a_2 \log^3(R), \tag{1}$$

   where T is in Kelvin, R is in Ohms, $a_0 = 1.129241 \times 10^{-3}$, $a_1 = 2.341077 \times 10^{-3}$, $a_2 = 8.775468 \times 10^{-8}$.

   Suppose that $R \in [1, 5]$ and is measured with the relative error $\delta(R) \leq 1.e-3$. To calculate $\log(R)$ you use your program `my_log`. Is it possible to calculate T with the absolute error $\Delta T \leq 2°K$? If possible, what is the maximal absolute error of $\log(R)$ calculation that can be specified in `my_log`? Explain.

<u>Attention:</u> You are given specifications for each of the programs to be submitted. We expect you to copy and use these spec files.