Introduction to Numerical Analysis, Air Force, Spring 2005

Assignment N4

Use singular value decomposition to solve two different problems:

I. Least squares problem in a ball

Write an m-function `x=lsq_ball(A,b,R)` that solves a least squares problem with a constraint:

$$\min_{\|x\| \le R} \|Ax - b\|.$$

Here $\|u\|$ is the 2-norm of a vector, A is an $m \times n$ matrix, b is an $m \times 1$ vector, R - a scalar (the ball radius). Suggested method:

1. Let $A = USV^T$ be svd of A and $k = rank(A)$. Then $\|Ax-b\| = \|U^T(Ax-b)\| = \|Sz-d\|$, where $z = V^Tx$ and $d = U^Tb$. Since $\|z\| = \|x\|$, we arrive at the problem

   $$\min_{\|z\| \le R} \|Sz - d\|$$

   with a diagonal matrix S or, equivalently,

   $$\min_{\sum_{i=1}^n z_i^2 \le R^2} \sum_{i=1}^k (g_i z_i - d_i)^2,$$

   where g is the diagonal of S, `g=diag(S)`.

2. The minimal norm solution of a corresponding problem without the constraint is $z_i = d_i/g_i$ for $i = 1, ..., k$ and $z_i = 0$ for $i = k+1, ..., n$. If this solution satisfies the constraint, $x = Vz$ is the needed solution. If $\|z\| > R$, the Lagrange multiplier method should be used. Derive a (nonlinear) equation $F(L, g, d, R) = 0$ for the Lagrange multiplier L and write an m-function `y=F(L,g,d,R)` needed to solve this equation numerically, e.g., using Matlab's function `fzero`. Your function `lsq_ball` should call `fzero` to find the Lagrange multiplier and then calculate the optimal point z. Again, $x = Vz$.

In your report describe briefly the usage of the Lagrange multiplier method and derive the equation for L. Submit your function `lsq_ball` and also the function `F(L,g,d,R)` it calls.

II. Data compression for black and white images

Write an m-function `im_compr(image,frmt,compr)` that reads an image file `image` written in format `frmt` using Matlab's function `X=imread(image,frmt)`, transforms the `uint8` array X into a matrix, `X=double(X)`, and then uses svd of X to compress the data in the ratios `compr(1), compr(2), ...` The program output consists of several graphs: the original image, a plot of the singular values of X, the images obtained after the data compression. Use `imagesc(X)` to draw the images and `colormap('gray')` (black and white images). In your report discuss the compression results for two high-resolution images from the NASA Photo Gallery of Asteroids, `gaspra.jpg` and `idasmoon.jpg` (The photos have been obtained from the Galileo spacecraft, the first planetary mission to photograph an asteroid "up-close". Its flyby of Gaspra occurred on 29 October 1991 at a distance of about 16,200 km. The second of the two asteroids which Galileo encountered en route to Jupiter, Ida was discovered to have something different: its own satellite! Galileo's flyby of Ida and its moon Dactyl occurred on 28 August 1993 at a distance of about 2,400 km.)