Assignment 1

1. Let $a > 0$ and b be known up to the absolute errors Δ_a and Δ_b, respectively. Estimate the absolute error of $y = a^b$ if powers are computed with the relative error δ. Assume $\Delta_a,\ \Delta_b$, and δ are small.

2. The Taylor series for the error function is

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}.$$

   This series converges for all x. Write an m-function `y=my_erf(x)` to evaluate $\operatorname{erf}(x)$ using this series. Use as many terms in the series as are necessary so that the first neglected term does not alter the accumulated sum when it is added to it in floating point. Since this is an alternating series, the error caused by truncating the infinite sum will then be less than the roundoff error. Investigate the effect of roundoff error by comparing the computed sum with the value obtained from Matlab's erf function. Try $x = 0.5,\ 1,\ 2,\ 3,\ 6$. Explain your results in your written report.

3. Evaluate

$$\sum_{k=1}^{\infty} \frac{1}{k^2+1}$$

   with the relative error not more than 10^{-8}. Make use of the following equalities: $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6},\ \ \sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$. Explain your calculations in your report.

4. For $|x| < 1$ we wish to calculate the expression

$$S(x) = \sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3+x}} - \sum_{k=1}^{\infty} \frac{1}{\sqrt{k^3-x}}.$$

   (a) Show that each series converges.
   (b) Approximately how many terms would it require to evaluate the first series with an error less than 10^{-8}?
   (c) Use some algebra to rewrite the expression $S(x)$ so that it can be evaluated more quickly.

5. Derive a stable recurrent method to calculate $S_n = \int_0^1 x^n \sin(ax)dx$ and $C_n = \int_0^1 x^n \cos(ax)dx$. Write an m-function `[Sn,Cn]=SC(n,a,err)` to find S_n and C_n with the absolute errors not exceeding *err* for $n,\ a$ given. In your report explain how the needed accuracy was achieved. *Hint*: you may assume roundoff errors are negligible and consider only the influence of an initial approximation error.