

# Critical State Modelling of Crossed Field Demagnetisation in HTS Materials

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**Abstract.** Large magnetic fields can be trapped in high temperature superconductor material, enabling their use as permanent magnets in various applications. It has, however, been observed that these materials experience a decay in magnetisation when a field is applied in a direction transverse to the trapped field. This paper presents the results of critical state modelling of an infinite slab of superconductor that is first fully magnetised and then exposed to magnetic field in the transverse direction. The bean and double critical state model are implemented and the results compared to each other and to that of physical experiment. The convergence of the decay is investigated as well as the effect of the magnitude of the crossed field on the final decay value. Diamagnetic and paramagnetic initial conditions are applied to the material and the results used to compare the accuracy of the bean and double critical state model.

## 1. Introduction

Bulk high temperature superconductors (HTS) possess the ability to trap large magnetic fields due to the phenomenon of flux pinning. This occurs when superconducting magnetic flux vortices, which move under the influence of external magnetic fields and the Lorentz force, become trapped at defects in the material crystal lattice called pinning centres. Field trapping thus enables the creation of high field magnets using bulk HTS materials. It has already been shown that fields as high as 2.5T can be obtained in single domain samples at 77K [1]. A range of possible applications exist for high field magnets, particularly in the design of high power density electric motors [2]. Before such devices can be created however, the characteristics of these trapped field magnets need to be well understood. A major cause for concern is the phenomenon of magnetisation decay, which has been observed when the material is subjected to magnetic fields in a direction transverse to the original magnetisation. This could possibly result in magnets losing their magnetisation and thus becoming worthless in applications which exploit the trapped field in HTS materials. The purpose of this paper is thus to evaluate the applicability of numerical modelling of HTS materials to further investigate this magnetisation decay. It compares results obtained with the Bean [3] and Double Critical State [4] model to that obtained by physical experiment. This comparison is used to evaluate which model would be most useful in future analysis of the crossed field demagnetisation of bulk HTS materials.

## 2. Underlying Equations

### 2.1. The Bean Model

The phenomenon of superconductivity can be described by critical state models which are based on observed physical properties of the material. They generally assume that critical current,  $J_c$ , flows in the critical state region. These models are usually formulated in terms of Maxwell's equation with added conditions governing the current density in the material. In the Bean model, this corresponds to a simplification of the extreme nonlinear  $\mathbf{E}$ - $\mathbf{J}$  relation such that if  $|J| < J_c$ , then  $E = 0$  and if  $|J| = J_c$ ,  $E \in 0, \infty$ . It is also assumed that  $\mathbf{E} \parallel \mathbf{J}$ .

When trying to find a numerical solution for the Bean model, the main difficulty encountered is in determining the unknown free boundary that separates the region that is in the critical state from the region that is not. Prigozhin [5] proposed a numerical solution of the Bean model by a variational formulation that treats the critical and non-critical region in the same way, thereby simplifying the solution. He derived a variational inequality that enables the solution of the current distribution in the material by determining the currents in the following set:

$$\mathbf{J} \in K_1 : \left( \mu_0 \mathcal{G} * \mathbf{J} + \frac{\partial \mathbf{A}_e}{\partial t}, \mathbf{J}' - \mathbf{J} \right) \geq 0 \quad (1)$$

where  $\mathcal{G} * \mathbf{J}$  represents the convolution of  $\mathbf{J}$  with the Green's function of Laplace's equation. The variational inequality can be solved by constrained optimisation with the constraints:

$$\begin{aligned} |\mathbf{J}| &< J_c \\ \mathbf{J}|_{t=0} &= \mathbf{J}_0 \\ \nabla \cdot \mathbf{J} &= 0 \end{aligned} \quad (2)$$

The simplest geometry that can be modelled easily for the purposes of studying the magnetisation decay phenomenon is the infinite slab in 1D. The variational formulation can be rewritten in terms of magnetic field for an infinite slab in the  $y - z$  plane, occupying the region  $0 \leq x \leq d$ . If it is assumed that the applied field is given by:  $\mathbf{H}_e(t) = (0, H_{ey}, H_{ez})$ , the induced current density, magnetic vector potential and total magnetic field (given by  $\mathbf{H}_e + \mathbf{h}$ ) will be parallel to the  $y - z$  plane. The advantage of rewriting the variational inequality in terms of magnetic field is that it avoids the computational expense of the convolution. If it is assumed that no external current is applied, the set of possible current densities is given by:

$$K = \left\{ \varphi(x) = (0, \varphi_y(x), \varphi_z(x)) \mid \int_0^d \varphi_y(x) dx = \int_0^d \varphi_z(x) dx = 0 \right\}$$

let  $\mathbf{J} \in K$  be current density, then:

$$\mathbf{J} = \nabla \times \mathbf{h}$$

which in the slab case corresponds to:

$$\begin{aligned} J_y &= -\frac{\partial h_z}{\partial x} \\ J_z &= \frac{\partial h_y}{\partial x} \end{aligned}$$

and

$$h_y = \int_0^x J_z(x) dx$$

$$h_z = - \int_0^x J_y(x) dx$$

which are zero at  $x = 0, d$ . If  $\mathbf{H} = \mathbf{H}_e + \mathbf{h}$ , then  $\mathbf{H} = \mathbf{H}_e$  at the boundary. A new set of possible induced magnetic fields is defined as:

$$K_h = \left\{ \psi(x) = (0, \psi_y(x), \psi_z(x)) \left| \begin{array}{l} \psi(0) = \psi(d) = 0 \\ |\nabla \times \psi| \leq J_c \text{ in } \Omega \end{array} \right. \right\}$$

$\text{curl } K_h \rightarrow K$  is a one to one map and hence,

$$\mathbf{h} \in K_{1h} = \left( \nabla \times \frac{\partial \mathbf{A}}{\partial t}, \nabla \times \psi - \nabla \times \mathbf{h} \right) \geq 0$$

with  $\mathbf{h}|_{t=0} = \mathbf{h}_0 \in K_h$ . Since  $\nabla \mathbf{A} = \mu_0(\mathbf{H}_e + \mathbf{h})$ , the inequality can be rewritten as:

$$\mathbf{h} \in K_{1h} : \mu \left( \frac{\partial \mathbf{h}}{\partial t} + \frac{\partial \mathbf{H}_e}{\partial t}, \psi - \mathbf{h} \right) \geq 0 \quad \forall \psi \in K_{1h} \quad (3)$$

with

$$K_{1h} = \left\{ \psi = (0, \psi_y, \psi_z) \left| \begin{array}{l} \psi(0) = \psi(d) = 0 \\ \left( \frac{\partial \psi_y}{\partial x} \right)^2 + \left( \frac{\partial \psi_z}{\partial x} \right)^2 \leq J_c^2 \end{array} \right. \right\}$$

which is solved in a 1D finite element scheme by constrained optimisation.

## 2.2. Double Critical State Model

The double critical state model is a modification of the Bean model designed to account for the instability of the magnetic flux vortices. The critical current is decomposed into two components,  $J_{c\parallel}$ , parallel to the magnetic flux vortices and  $J_{c\perp}$ , perpendicular to the vortices. The main assumption of the double critical state model is that  $J_{c\perp}$  is equal to the critical current required to depin the flux vortices, while  $J_{c\parallel}$  is the critical current at which vortex instability occurs. The definition of the double critical state model means that the variational formulation for the infinite slab problem will be exactly the same as that for the bean model, with only the constraints on the current density changing, thus:

$$K_{1h} = \left\{ \psi = (0, \psi_y, \psi_z) \left| \begin{array}{l} \psi(0) = \psi(d) = 0 \\ |(\nabla \times \psi) \times \mathbf{H}| \leq J_{c\perp} |\mathbf{H}| \\ |(\nabla \times \psi) \cdot \mathbf{H}| \leq J_{c\parallel} |\mathbf{H}| \end{array} \right. \right\}$$

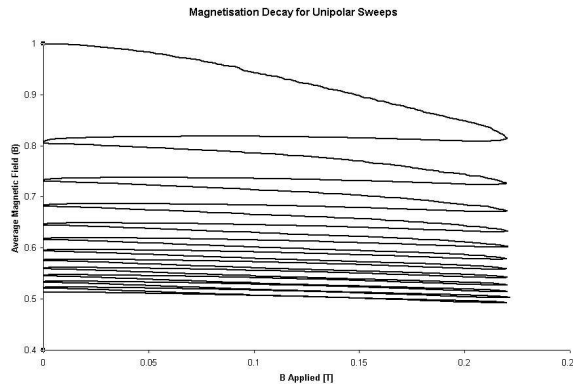
with  $\mathbf{H} = \mathbf{H}_e + \mathbf{h}$  and:

$$|(\nabla \times \psi) \times \mathbf{H}| = \left| H_y \frac{\partial \psi_y}{\partial x} + H_z \frac{\partial \psi_z}{\partial x} \right|$$

$$|(\nabla \times \psi) \cdot \mathbf{H}| = \left| -H_y \frac{\partial \psi_z}{\partial x} + H_z \frac{\partial \psi_y}{\partial x} \right|$$

### 3. Results

Vanderbemden [6] performed an experiment where he applied a train of crossed field pulses to an initially magnetised single domain YBCO sample. He found that the crossed field caused a decay in the trapped field. This field decay is shown in figure 1 for a unipolar crossed field.



**Figure 1.** Average  $y$  component of magnetic field for crossed field experiment

He also found that bipolar crossed field pulses caused a greater decay in the trapped field magnetisation than a unipolar applied field. Similar results have been reported by other researchers conducting experiments involving crossed fields. Vanderbemden further investigated whether the decay in magnetisation converges to a steady state value. He found no convergence after 100 crossed field sweeps although other researchers claim convergence [7].

The Bean and Double Critical State models have been implemented here and their applicability evaluated by comparison to the results of Vanderbemden's crossed field experiment.

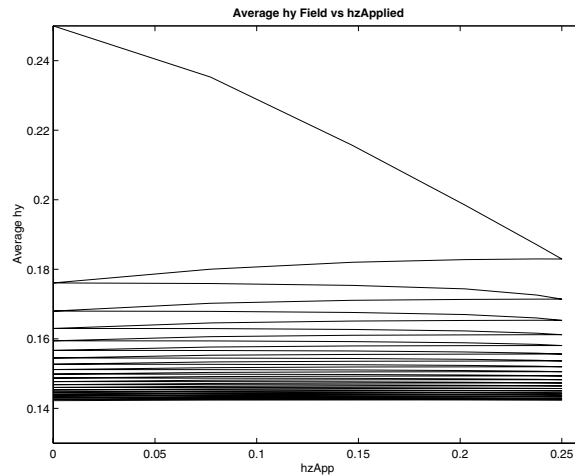
#### 3.1. Bean Model Results

The slab is initially magnetised by applying a single pulse of twice the magnitude field required to fully penetrate the material. This ensures that the maximum field is trapped in the superconductor. A train of crossed field pulses of smaller magnitude is then applied and the magnetisation decay is plotted as the average  $H_y$  component magnetic field with respect to the sweeping crossed field  $H_z$ . Parameter values of  $J_c = 1$ ,  $d = 1$  and  $\mu_0 = 1$  are chosen for convenience.

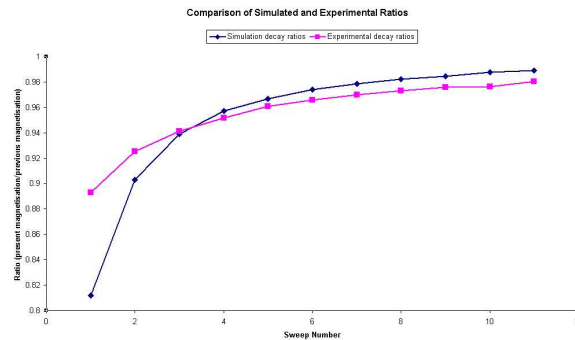
The crossed field simulation for the Bean model, shown in figure 2, produced results showing a magnetisation decay similar to that of the physical experiment. Here the  $y$  component of the average magnetic field is plotted against the sweeping crossed field  $h_z$ .

The Bean model curve shows a much larger decay in magnetisation after the first crossed field sweep than the experimental decay curve. Both curves have a concave down curvature and the  $h_y$  field decreases for each successive crossed field sweep. The bean model does not show the crossing over observed with the experimental curve. To compare the two curves on a similar scale, the ratio of the average field after each sweep with respect to the previous sweep was plotted for both curves and the results plotted in figure 3.

The large disagreement in the magnetisation decay after the initial crossed field sweep is confirmed, but the rest of the decay characteristic shows good agreement, with the largest difference between the two decay rates being 0.2%. The Bean model also showed a similar



**Figure 2.** Simulation result of crossed field experiment using Bean model



**Figure 3.** Comparison of magnetisation decay rates of simulation and experiment

trend to the experimental results with a greater decay in magnetisation being caused by bipolar crossed fields than the unipolar fields.

The convergence of the magnetisation decay was investigated to determine whether a trapped field magnet would lose all its magnetisation when subjected to crossed magnetic fields or just a fraction. A further simulation was run to verify this convergence. The simulation was set up with a crossed field performing 5000 sweeps. This was compared to a simulation with 20 crossed field sweeps. In both simulations, the magnetisation decay converged to a non-zero value within the same defined tolerance range. This suggests that not only does the decay converge, but that it also reaches its convergence point quite rapidly.

The dependence of this final decay value on the magnitude of the crossed field was investigated by running further simulations. Figure 5 shows the results of this investigation. These results show that the magnetisation loss is low for low values of crossed field. This result is encouraging because the magnitude of the crossed field expected in a typical application would likely be low compared to the magnitudes needed for full penetration of the material.

The differences observed between the numerical and experimental results could most likely be attributed to the following factors:

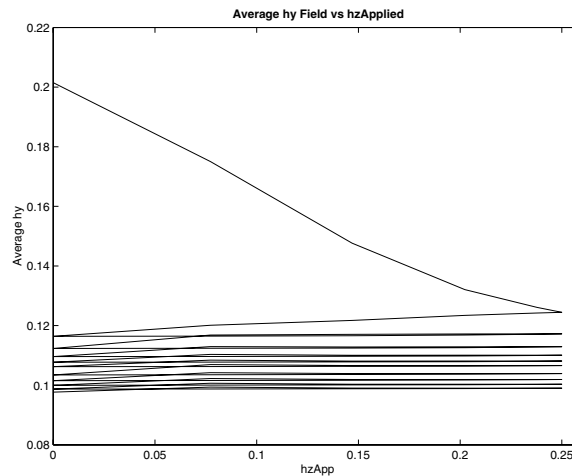
- An infinite slab was modelled in the numerical simulation, while the physical experiment was conducted on a finite domain cylindrical shape puck. The effect of the difference in

geometry is not taken into account in the simulation.

- The simulation ignores material anisotropy. Physically, this anisotropy affects the critical current density in different directions in the material and could well have an effect on the magnetisation decay measured.
- The Bean model assumes that  $J_c$  is constant while it has been shown experimentally that  $J_c$  is magnetic field dependent.
- The effect of temperature is ignored in the numerical simulation.

### 3.2. Double Critical State Model Results

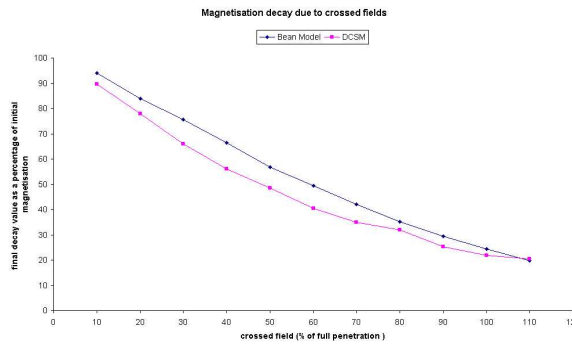
The same crossed field experiment was simulated using the DCSM code with  $J_{c\parallel} = 0.6$  and  $J_{c\perp} = 0.8$ , and once again a decay in  $y$  direction average magnetic field due to  $z$  direction field pulsing was observed as shown in figure 4.



**Figure 4.** Simulation result of crossed field experiment using Double Critical State model

Although a similar pattern decay exists, the shape of the magnetisation curve is quite different. The magnetisation curve for the DCSM is concave up for the first half of the first sweep in contrast to the experimental results. This concavity does however change to concave down as the magnitude of the crossed field is increased to a value corresponding to full penetration. Subsequent cross field sweeps cause the curve to cross itself at various points. The crossing here is somewhat more pronounced than the experimentally measured curve. It can also be seen that the DCSM solution magnetisation curve displays an irregular magnetisation decay where the average  $h_y$  does not decrease by smaller amounts after each sweep, but by seemingly random amounts. The DCSM addresses the Bean model assumption that the current density is always perpendicular to the flux vortices. It incorporates a component of  $J_c$  parallel to the flux vortices to account for vortex instability. This model however suffers from the same shortcomings as the Bean model in that it does not account for the magnetic field dependence of critical current or the effect of temperature. It is also subject to the same error due to the different geometry which is modelled as opposed to that of the experiment.

The DCSM appears to show the same convergence of magnetisation decay to a non-zero value when run with 100 crossed field sweeps. A comparison of the final decay value obtained for the Bean Model and DCSM was performed and the results shown in figure 5.



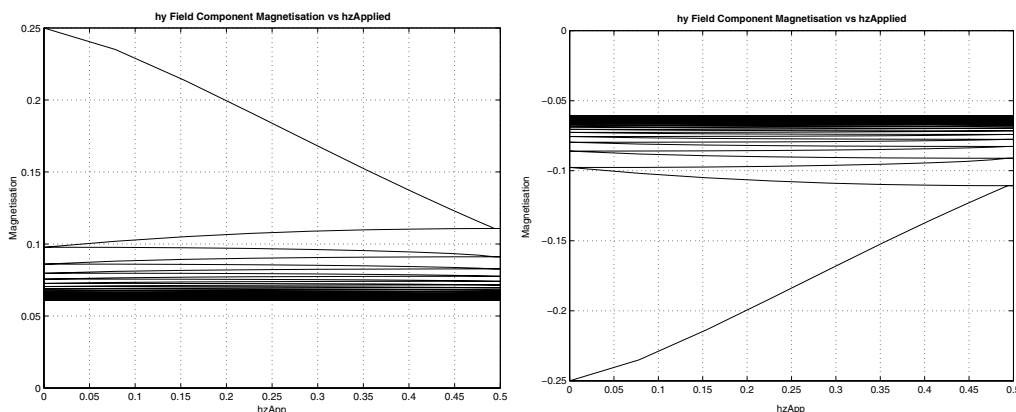
**Figure 5.** Comparison of final decay value found for Double Critical State model and Bean Model

*3.3. Diamagnetic and Paramagnetic Response*

Fisher [7] conducted an experiment in which he applied crossed fields to superconductors with trapped magnetic fields. He used two differently shaped samples. One was a YBCO plate and the other was a hexagonal single crystal of YBCO. Two distinct cases were tested: the paramagnetic initial condition and the diamagnetic initial condition. He found that the response was symmetrical for these cases. The concavity of the magnetisation decay curves varied between the two samples lending proof to the theory that the geometry of the sample has an effect on the demagnetisation characteristic. This would explain some of the discrepancies seen between the models developed here and the experimental results.

To further examine the applicability of these critical state models, it is thus desirable to duplicate this experiment to determine whether the response is symmetric in each of the Bean and Double critical state model.

Figure 6 shows the response of the material to these initial conditions for the Bean model and figure 7 shows the response for the Double Critical State model with full penetration crossed field applied. It can clearly be seen that the Bean model shows a symmetrical response while the double critical state model shows an asymmetrical response in contrast to the experimental results of Fisher.



**Figure 6.** Paramagnetic and Diamagnetic response for Bean Model

Fisher also tested his results with the Double Critical state model and found a similar asymmetry in its response. The Bean model however gives the symmetrical response sought

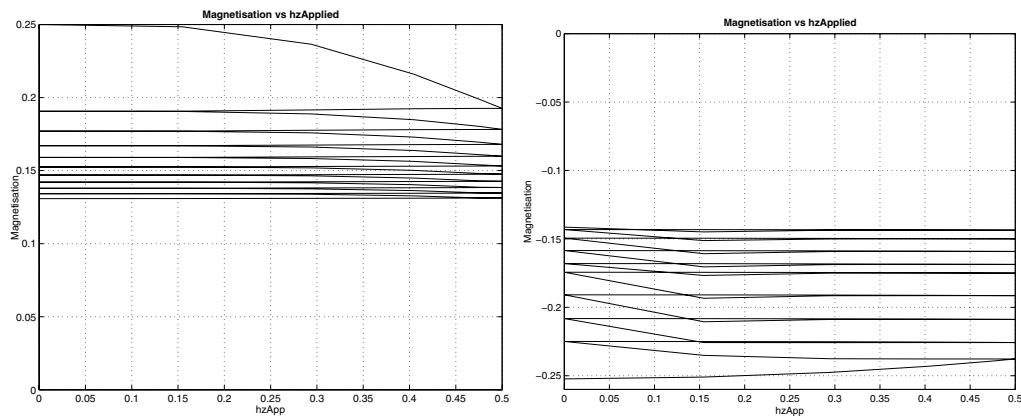


Figure 7. Paramagnetic and Diamagnetic response for Double Critical State Model

and it is therefore concluded that for the purposes of investigation of magnetisation decay, the Bean model is the better model of the two.

#### 4. Conclusion

Critical state models provide a convenient way to model bulk HTS materials when investigating the magnetisation phenomenon. The Bean model in particular has shown good agreement with experimental results. It confirms that a decay in trapped field magnetisation occurs due to crossed magnetic fields and that the decay is more severe for bipolar crossed fields than for unipolar crossed fields. Simulations involving a large number of crossed field sweeps show that the magnetisation decay converges to a non-zero value and that this value is dependent on the magnitude of the crossed field. The Bean model has been shown to more accurately model the experimental results because it shows a symmetrical decay for both the paramagnetic and diamagnetic initial condition whereas the Double Critical State model gives an asymmetrical response.

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