(1) Correct. If $V \subseteq Y$ is open then $f^{-1}(V) = (f \restriction U_1))^{-1}(V) \cup (f \restriction U_2)(V)$ and hence open.
(2) Correct. Let $x_0 \in X$ be any point an let

$$Y = \{x \in X : x \text{ is connected by a path to } x_0 \}.$$

Now Y is nonempty, open and its complement is also open, hence $Y = X$.
(3) Correct. The closed pentagon is hemeomorphic to the closed unit disk. So Brower's fixed point theorem applies.
(4) False. An infintie discrete space is metric of radius 1 and every point is isolated, but is not compact.
(5) Correct. As every U_n contains the rationals, the intersections $\bigcap_n U_n$ is a dense G_δ subset of the complete metric space R. By the Baire Category theorem it cannot be countable.
(6) Part B: this is easier than the proof in Ex. 8 that there are no convergin sequences at all. You only need to prove that there is no converging sequence of *principal* ultrafilters to a non-principal one. If F_m denotes the principal ultrafilter $\{X : n \in X \subseteq \mathbb{N}\}$ and we assume to the contrary that $\langle F_{m_n} : n \in \mathbb{N}\rangle \to F\rangle$, F nonprincipal, then by thinning out we may assume the sequence $\langle m_n : n \in \mathbb{N}\rangle$ is 1-1 (it is not eventually constant because F is not principal). Let $A = \{m_{2n} : n \in \mathbb{N}\}$. So exactly one of the infinite sets A or $\mathbb{N} \setminus A$ belongs to F and infinitely many F_n remain outsided the basic clopen neighborhood of F determined by this set. Contradiction to convergence.
(7) All was done in class.
(8) Part B: The open unit interval and the real line are homeomorphic. The completion of the former is compact and the completion of the latter is not compact.

Part C: The functions f_n embed X into a countable product of unit intervals via the evaluation map $x \mapsto \langle f_n(x) : n \in X\rangle$. Now the closure of the image in $[-1, 1]^{\mathbb{N}}$ is a closed subset of a compact metric space.