On supercompact spaces - Abstract

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A topological space X is *supercompact* [1] if it has a binary subbase for the closed sets. A collection of sets $\mathcal{B}$ is *binary* if for every $\mathcal{C} \subseteq \mathcal{B}$ with $\bigcap \mathcal{C} = \emptyset$ we can find $C_0, C_1 \in \mathcal{C}$ with $C_0 \cap C_1 = \emptyset$. A natural example of a supercompact space is the closed unit interval $[0,1]$ with the subbase $\{[a,b] \colon 0 \leqslant a < b \leqslant 1\}$. By Alexander's Lemma, every supercompact space is compact. The converse is not true: an infinite supercompact space always contains convergent sequences. Moreover, there exist first-countable compact spaces which are not supercompact, see [2]. Strok & Szymański proved in [3] that metric compact spaces are supercompact.

We will give a survey of known results and open problems on supercompact spaces. We are interested in the class of *normally supercompact spaces*. A space X is normally supercompact if it has a binary closed subbase $\mathcal{B}$ which is a *normal family*, i.e. $\{\{x\} \colon x \in X\} \subseteq \mathcal{B}$ and for each disjoint sets $A, B \in \mathcal{B}$ there exist $A', B' \in \mathcal{B}$ such that $A' \cap B = \emptyset = A \cap B'$ and $A' \cup B' = X$. This class of spaces is strictly smaller than the class of all supercompact spaces, even in the metrizable case. We show that a retract of any Cantor cube is normally supercompact. The problem whether, in general, a retract of a supercompact space is supercompact, is open. Finally, we will show that a normally supercompact space with a point of uncountable regular character κ either contains the one-point compactification of the discrete space of size κ or else contains a copy of $\kappa + 1$.

References

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