# Cluster-Based Algorithms for Relational Join 

The New Computation Model
More Efficient Joins Via Replication Optimum Strategies for Special Cases

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## Next-Generation File Systems

Files are very large.
-They are divided into chunks.

- Perhaps 16MB to a chunk.

Chunks are replicated at several
compute-nodes.
A master (possibly replicated) keeps track of all locations of all chunks.

## Compute Nodes

-Organized into racks.

- Intra-rack connection typically gigabit speed.
- Inter-rack connection faster by a small factor.



## Racks of Compute Nodes

File


Chunks


## 3-way replication of files, with copies on different racks.

## Implementations

-GFS (Google File System - proprietary).
$\checkmark$ HDFS (Hadoop Distributed File System open source).
-KFS (Kosmix File System). Clustera (Implementation at U. Wisconsin).

## Moving Up the Computation Stack

BigTable (Google retrieval system for objects).
-Hadoop (Open-source implementation of Google's map-reduce).
-PIG (Yahoo! implementation of relational algebra/SQL on top of Hadoop).

## Reading and Writing Files

Order of elements in a file doesn't matter.

- Example: A file could be a collection of documents; order within a doc matters, but not the order of docs.
Parallel reading, writing OK.


## Algorithms

Algorithm = acyclic collection of processes.

- Arc means the process at the tail feeds some data to the head.


## Example Algorithm



## Communication Cost

- Communication cost $=$ sum of sizes of inputs to all processes of an algorithm.
- Also, elapsed communication cost: the maximum over all paths through the acyclic graph of the sizes of the inputs to each of the processes on that path.


## Why Not Count Output Size?

Outputs of one process are inputs to at least one other, or are algorithm output.
Algorithm outputs tend to be small because users can't make use of too much information.

## Why Not Count Processing Time?

- In the types of applications we discuss, work by a process is usually proportional to input size.
$\rightarrow$ And it occurs in main memory, so we can do a lot in the time it takes to get your input over a gigabit line.


## Natural Join of Relations

-Given: a collection of relations, each with attributes labeling their columns.

- Find: Those tuples over all the attributes such that when restricted to the attributes of any relation $R$, that tuple is in $R$.


## Example: Natural Join



## Map-Reduce Algorithms

- Map processes send inputs to keyvalue pairs. - "keys" are not necessarily unique.

Outputs of Map processes are sorted by key, and each key is assigned to one Reduce process.
Reduce processes combine values associated with a key.

## Joining by Map-Reduce

-Suppose we want to compute $\mathrm{R}(\mathrm{A}, \mathrm{B})$ JOIN $\mathrm{S}(\mathrm{B}, \mathrm{C})$, using $k$ compute nodes.
$-R$ and $S$ are each stored in a chunked file.

## Joining by Map-Reduce - (2)

-Use a hash function $h$ from B-values to $k$ buckets.

- Many Map processes take chunks from $R$ and $S$, and send:
- Tuple $\mathrm{R}(\mathrm{a}, \mathrm{b})$ to Reduce process $\mathrm{h}(\mathrm{b})$.
- Tuple S(b,c) to Reduce process h(b).


## Joining by Map-Reduce - (3)

$\rightarrow$ If $\mathrm{R}(\mathrm{a}, \mathrm{b})$ joins with $\mathrm{S}(\mathrm{b}, \mathrm{c})$, then both tuples are sent to Reduce process $\mathrm{h}(\mathrm{b})$.
-Thus, their join ( $\mathrm{a}, \mathrm{b}, \mathrm{c}$ ) will be produced there and shipped to the output file.

## 3-Way Join

Consider a chain of three relations: R(A, B) JOIN S(B, C) JOIN T(C,D)

- Example: R, S, and T are "friends" relations.
-We could join any two by the 2-way map-reduce algorithm shown, then join the third with the resulting relation.
But intermediate joins are large.
3-Way Join - (2)
- An alternative is to divide the work among $k=m^{2}$ Reduce processes.
$\Delta$ Hash both B and C to $m$ values.
- A Reduce process corresponds to a hashed $B$-value and a hashed $C$-value.
3-Way Join - (3)

Each S-tuple $S(b, c)$ is sent to one Reduce process: (h(b), h(c)).
But each tuple R(a,b) must be sent to $m$ Reduce processes (h(b), x).
And each tuple T(c,d) must be sent to $m$ Reduce processes ( $\mathrm{y}, \mathrm{h}(\mathrm{c})$ ).

## Example: $m=4 ; \mathrm{k}=16$.


$T(c, d)$
3-Way Join - (4)

Thus, any joining tuples $R(a, b), S(b, c)$, and $T(c, d)$ will be joined at the Reduce process (h(b), h(c)).
Communication cost: s + mr + mt.

- Convention: Lower-case letter is the size of the relation whose name is the corresponding upper-case letter.
- Example: $r$ is the size of $R$.


## Comparison of Methods

-Suppose for simplicity that:

- Relations R, S, and T have the same size $r$.
- The probability of two tuples joining is $p$.

The 3-way join has cost $\mathrm{r}(2 \mathrm{~m}+1)$.

- Two two-way joins have a cost of:
- $3 r$ to read the relations, plus
- $\mathrm{pr}^{2}$ to read the join of the first two.
- Total = r(3+pr).


## Comparison - (2)

3-way beats 2 -way if $2 m+1<3+p r$. $p r$ is the multiplicity of each join. - Thus, the 3-way chain-join is useful when the multiplicity is high.
Example: relations are "friends"; $p r$ is about 300. $m^{2}=k$ can be 20,000.
Example: relations are Web links; pr is about 15. $m^{2}=k$ can be 64 .

## Some Questions

When we discussed the 3-way chainjoin, we used attributes B and C for the map-key (index for the Reduce processes).
Why not include A and/or D?
Why use the same number of buckets
for $B$ and $C$ ?

## Share Variables

$\rightarrow$ For the general problem, we use a share variable for each attribute.

- The number of buckets into which values of that attribute are hashed.
Convention: The share variable for an attribute is the corresponding lowercase letter.
- Example: the share variable for attribute A is always $a$.


## Share Variables - (2)

The product of all the share variables is
$k$, the number of Reduce processes.

- The communication cost of a multiway
join is the sum of the size of each relation times the product of the share variables for the attributes that do not appear in the schema of that relation.


## Example: Minimizing Cost

-Consider the cyclic join R(A, B) JOIN S(B, C) JOIN T(A, C)
Cost function is rc + sa + tb.
Construct the Lagrangean:
$r c+s a+t b-\lambda(a b c-k)$
$\rightarrow$ Take derivative wrt each share variable, then multiply by that variable.

- Result is 0 at minimum.


## Example - Continued

$-d / d a$ of rc $+s a+t b-\lambda(a b c-k)$ is $s-\lambda b c$.
$\rightarrow$ Multiply by $a$ and set to $0: s a-\lambda a b c=0$. Note: $a b c=k: s a=\lambda k$. Similarly, $\mathrm{d} / \mathrm{db}$ and $\mathrm{d} / \mathrm{dc}$ give: $s a=t b=r c=\lambda k$.
Solution: $a=\left(k r t / s^{2}\right)^{1 / 3} ; b=\left(k r s / t^{2}\right)^{1 / 3}$; $c=\left(k s t / r^{2}\right)^{1 / 3} ;$
Cost $=r c+s a+t b=3(k r s t)^{1 / 3}$.

## Dominated Attributes

-Certain attributes can't be in the mapkey.
A dominates B if every relation of the join with $B$ also has $A$.

- Example:
$R(A, B, C)$ JOIN $S(A, B, D)$ JOIN T $(A, E)$ JOIN $U(C, E)$
Every place with B
Also has $A$


## Example - (2)

R(A,B,C) JOIN S(A,B,D) JOIN T(A,E) JOIN U(C,E)

- Cost expression:
rde + sce + tbcd + uabd
Since $b$ appears wherever $a$ does, if there were a minimum-cost solution with $b>1$, we could replace $b$ by 1 and $a$ by $a b$, and the cost would lower.


## Dominated Attributes - Continued

Thus, we do not put any dominated attribute in the map-key.
-This rule explains why, in the discussion of the chain join
R(A, B) JOIN S(B, C) JOIN T (C,D) we did not put $A$ or $D$ in the map key.

## Solving the General Case

- Unfortunately, there are more complex cases than dominated attributes, where the equations derived from the Lagrangean imply a positive sum of several terms $=0$.
- We can fix, generalizing dominated attributes, but we have to branch on which attribute needs to be eliminated from the map-key.


## Solving - (2)

Solutions not in integers:

- Drop an attribute with a share < 1 from the map-key and re-solve.
- Round other nonintegers, and treat $k$ as a suggestion, since the product of the integers may not be $k$.


## Special Case: Star Joins

- A star join combines a large fact table $\mathrm{F}\left(\mathrm{A}_{1}, \mathrm{~A}_{2}, \ldots, \mathrm{~A}_{n}\right)$ with tiny dimension tables
$\mathrm{D}_{1}\left(\mathrm{~A}_{1}, \mathrm{~B}_{1}\right), \mathrm{D}_{2}\left(\mathrm{~A}_{2}, \mathrm{~B}_{2}\right), \ldots, \mathrm{D}_{n}\left(\mathrm{~A}_{n}, \mathrm{~B}_{n}\right)$.
- There may be other attributes not shown, each belonging to only one relation.
Example: Facts = sales; dimensions tell about buyer, product, etc.


## Star-Join Pattern



## Star Joins - (2)

- Map-key = the A's.
- B's are dominated.

Solution: $d_{i} a_{i}=\lambda k$ for all $i$.

- That is, the shares are inversely proportional to the dimension-table sizes.


## Cool Application of Star Join Result

-Fact/dimension tables are often used for analytics.

- Example: fact table is all sales records; dimension tables give info about customers, products, suppliers, etc.
Aster Data approach: partition fact table among nodes permanently; replicate needed pieces of dimension tables.


## Star-Join Application - (2)

Our solution lets you partition the fact table to $k$ nodes.
$\rightarrow$ Replication of tuples in the dimension tables is minimized.

## Chain Joins

$\rightarrow$ A chain join has the form
$R\left(A_{0}, A_{1}\right)$ JOIN R $\left(A_{1}, A_{2}\right)$ JOIN ... JOIN R( $\left.A_{n-1}, A_{n}\right)$

- Other attributes may appear, but only in one relation.
$\bullet \mathrm{A}_{0}$ and $\mathrm{A}_{n}$ are dominated; other attributes are in the map-key.



## Special Case: All Relations Have the Same Size

Illustrates strange behavior.

- Even and odd $n$ have very different distributions of the share variables.
$\checkmark$ Even $n: a_{2}=a_{4}=\ldots=a_{n-2}=1$;

$$
a_{1}=a_{3}=\ldots=a_{n-1}=k^{2 / n}
$$

## Pattern for Even n



## Odd n, Equal Relation Sizes

Even a's grow exponentially.

- That is, $a_{4}=a_{2}{ }^{2} ; a_{6}=a_{2}{ }^{3} ; a_{8}=a_{2}{ }^{4}, \ldots$
- The odd a's form the inverse sequence.
- That is, $a_{1}=a_{n-1} ; a_{3}=a_{n-3} ; a_{5}=a_{n-5} ; \ldots$


## Pattern for Odd $n$



## Summary

1. Multiway joins can be computed by replicating tuples and distributing them to many compute nodes.
2. Minimizing communication requires us to solve a nonlinear optimization.
3. Method wins for star queries and queries on high-fanout graphs.
4. Exact solution for chain and star queries.
