Non-cooperative Scheduling and Power Control in Wireless Collision Channels

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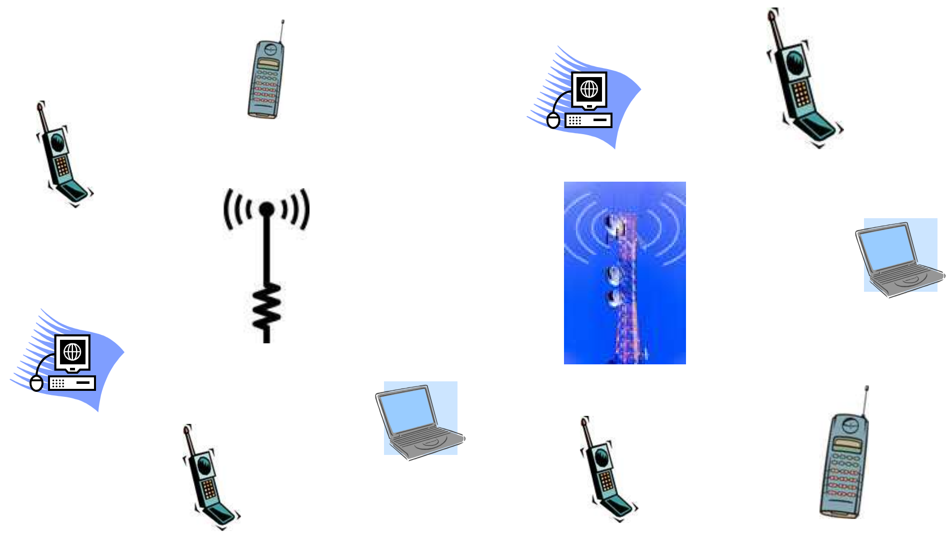
Background

- Game-theoretic analysis of (wireless) communication networks has been of much recent interest.
- The general setting we consider is that of independent users, who seek to optimize their personal utility, subject to system constraints.
- Motivation: Avoid centralized control where possible.



Analysis Framework

- Determine utility function for each user ("player")
- Establish properties of the Nash Equilibrium Point (NEP): Existence, uniqueness, computation, efficiency ("price of anarchy").
- Validity of the NEP* (stability, convergence of reasonable§ distributed algorithms)
- Mechanism design: Making the NEP more efficient.

* Aumann (2005): When All is Said and Done, How Should You Play and What Should You Expect?

§ Y. Shoham (2007): If Multi-agent Learning is the Answer, What is the Question?

Related Applications

The game theoretic framework has been applied, among many other applications, to:

- Selfish routing
- Distributed power control in CDMA networks
- Spectrum assignment is cognitive-radio networks
- Transmission scheduling in collision channels

Problem Description

- We consider a wireless collision channel, connecting mobile users (*mobiles*) to a common base station.
- Users schedule their transmissions in a distributed manner.
- Each user wishes to minimize its average power investment, subject to a required throughput constraint.
- The model allows for:
  - Asymmetric users
  - Time-varying channel conditions (fading channels)
  - Power level control

Related Literature on Collision Channels

- Decentralized random access mechanisms in collision channels (Aloha and its variants) have been extensively studied. These are currently utilized, e.g. in 802.11.
- Non-cooperative analysis was considered in several recent papers, including McKenzie & Wicker (2003), Altman, Azouzi & Jimenez (2004), Inaltekin & Wicker (2006), Altman, Avrachenko, Miller & Prabhu (2007).
- In this work we shall focus on the concept of fixed-rate equilibrium (previously considered, but not analyzed, in Jin & Kesidis, 2002).

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Basic Channel Model

- We consider a time-slotted collision channel, shared by a finite number $\{1,\ldots,n\}$ of users.
- Thus, if two or more users transmit at the same slot, their data is lost.
- Let R_j denote user j's effective throughput per successful transmission.

User Model

- Each user j is assigned a nominal throughput demand d_j, which needs to be transmitted to the base station.
- The throughput demand d_j may be:
  - Negotiated and authorized by the base station (in which case is may serve as a management tool).
  - Determined by the user itself, according to its application.
- Each user is responsible for scheduling its own transmissions. The user's goal is to attain its required throughput with minimal power investment.

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Transmission Strategies

- We start by assuming that:
  - Each user is employing a stationary transmission strategy: Transmit at each slot with p_j. Denote $\mathbf{p} = (p_{1,\dots,}p_n)$.
  - Each user has a single power level available (hence – no power control).
- Assuming that users always have packets to send (*saturated buffer*), the expected throughput per slot for user j is:

$$r_j(\mathbf{p}) = R_j p_j \prod_{i \neq j} (1 - p_i)$$

- The user's throughput requirement is $r_j(\mathbf{p}) \geq d_j$. Under minimal power objective, this reduces to: $r_j(\mathbf{p}) = d_j$.

The Equilibrium Equations

- Summarizing, we obtain the following equilibrium equations:

$$R_j p_j \prod_{i \neq j} (1 - p_i) = d_j \,, \quad j = 1, \ldots n$$

Any vector $\mathbf{p} = (p_1, \ldots, p_n)$ of transmission probabilities that satisfies these is called an equilibrium point.

- Note: When all other users employ stationary strategies, then a stationary strategy will be a best-response for a user among all strategies. Hence, an equilibrium point is stationary strategies is an equilibrium in general strategies.

Fading Channels

- Assume next that the quality of the "free channel" of each user changes in time. We model this variation as a stationary and ergodic stochastic process.
- Further assume that channel variation of different users is independent.
- At the beginning of every slot *k*, each user j obtains a Channel State Information (CSI) signal $z_{jk} \in Z_j$ that signals the channel state for the next slot. The user may adapt its coding scheme accordingly.
- As a consequence, the expected throughput of a successful transmission in that slot changes to $R_j(z_{jk})$.

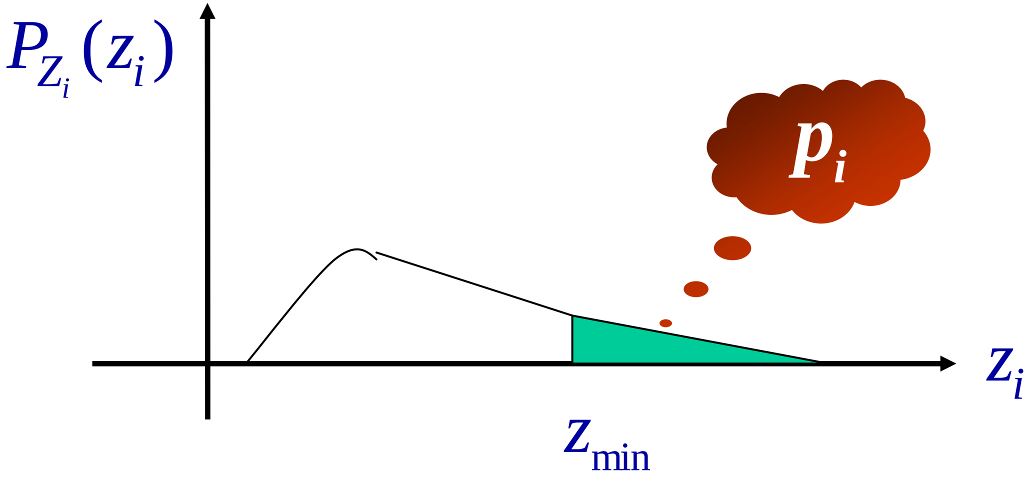
Fading Channel: Modified Transmission Strategies

- We now allow the transmission decision of each user to depend (only) on its own CSI for the current slot. We refer to such strategies as locally stationary strategies (LSS). Thus, an LSS is a mapping $\sigma_j : Z_j \to [0,1]$.

- It may be seen that a best-response strategy of user j is always a threshold strategy: Transmit only in better channel conditions, down to some threshold for which the throughput demand d_j is achieved.

Threshold Strategies

- This reduction to threshold strategies allows to define a 1-1 mapping $\bar{R}_j = H_j(p_j)$ between the average transmission probability $p_j = E\{\sigma_j(z_j)\}$ and the average throughput per slot, $\bar{R}_j = E\{\sigma_j(z_j)R_j(z_j)\}$. Further, the function $H_j(p_j)$ is increasing and concave in p_j.



Fading Channel: Modified Equilibrium Equations

- We now obtain the modified equilibrium equations:

$$H_j(p_j)\prod_{i\neq j}(1-p_i)=d_j\,,\quad j=1,\dots n$$

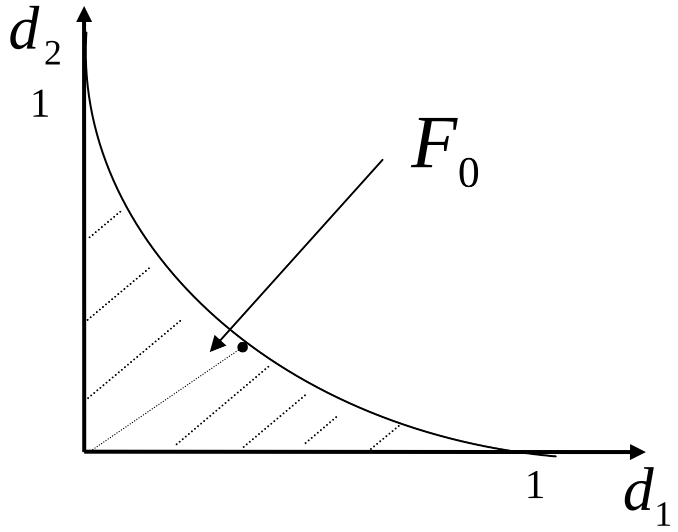
where each function $H_j(p_j)$ is concave increasing.

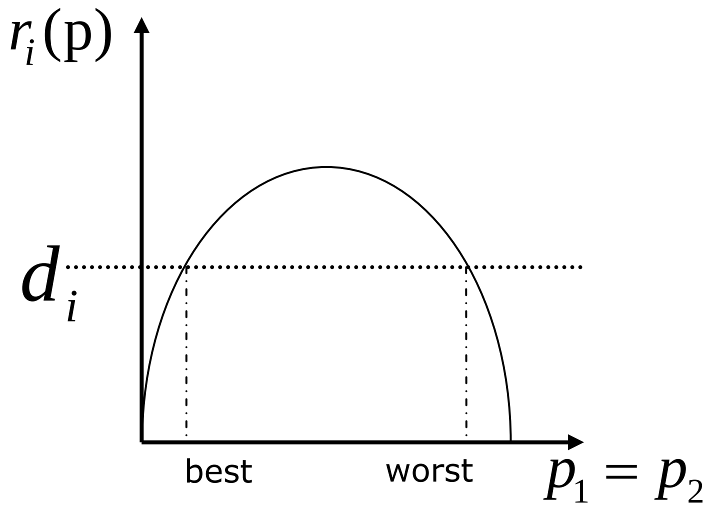
Main Results

- Let F_0 denote the feasible region of throughput demand vectors $(d_1, \dots d_n)$ for which there exists (at least one) equilibrium point. F_0 is a non-empty, closed and bounded set.

- Two equilibria or none: For any demand vector $(d_1, \dots, d_n)$ inside F_0, there exist two equilibrium points, say p^* and p^o.

- Of these two equilibrium points, one is uniformly better than the other. That is, $p_j^* < p_j^o$ holds for each user $j = 1, \dots, n$.





Efficiency

We next examine the efficiency of the above equilibrium points, in terms of total power of all users.

- The better equilibrium p^* coincides with the social optimum (in locally stationary strategies).
  Hence: "Price of Anarchy" =1.
- The worse equilibrium p^o can be arbitrarily worse than the social optimum. Hence: "Price of Stability"$\to\infty$.

Note: In the many-user limit, the worst-case channel capacity (no CSI, symmetric users) approaches e^{-1}.

Distributed Convergence to Equilibrium

- Given the existence of a better and worse equilibrium, it is evidently of common interest to devise a mechanism that converges to the better equilibrium.
- Consider the following best-response dynamics:
  - Each user j monitors the current channel usage, namely

$$U_j = \prod_{i \neq j} (1 - p_i).$$

  - Once in a while the user adjusts its transmission strategy to maintain its throughput demand d_j according to

$$H(p_j)U_j = d_j, \quad \text{or} \quad p_j = H^{-1}(d_j / U_j)$$

Convergence of Best-Response Dynamics

Theorem: Suppose that

(a) "Slow start": The initial transmission probabilities of all users are smaller than the better equilibrium: $\mathrm{p}(0) \leq \mathrm{p}^*$.

(b) Each mobile updates its strategy infinitely often

Then the sequence $\mathbf{p}(t)$ is monotonously increasing, and converges to the better equilibrium p^*.

- Note that some requirement on the initial conditional is necessary: If we start at the "wrong" equilibrium, we simply stay there.
- Additional results: Resilience to joining and leaving users.

Power-level Control

- Suppose that each user can now transmit at one of several power levels.
- The power level S_j affects only the free-channel throughput, but not the collision event:

$$R_j = R_j(z_j, S_j)$$

- It turns out that the best-response strategy of each user follows a water-filling power allocation scheme. Using this reduction, we can still obtain the equilibrium equations in the familiar form:

$$H_j(p_j)\prod_{i \neq j}(1 - p_i) = d_j\ , \quad j = 1, \ldots n$$

The caveat: Now $H_j(p_j)$ may be non-concave.

Power-level Control: Main Results

As Before:

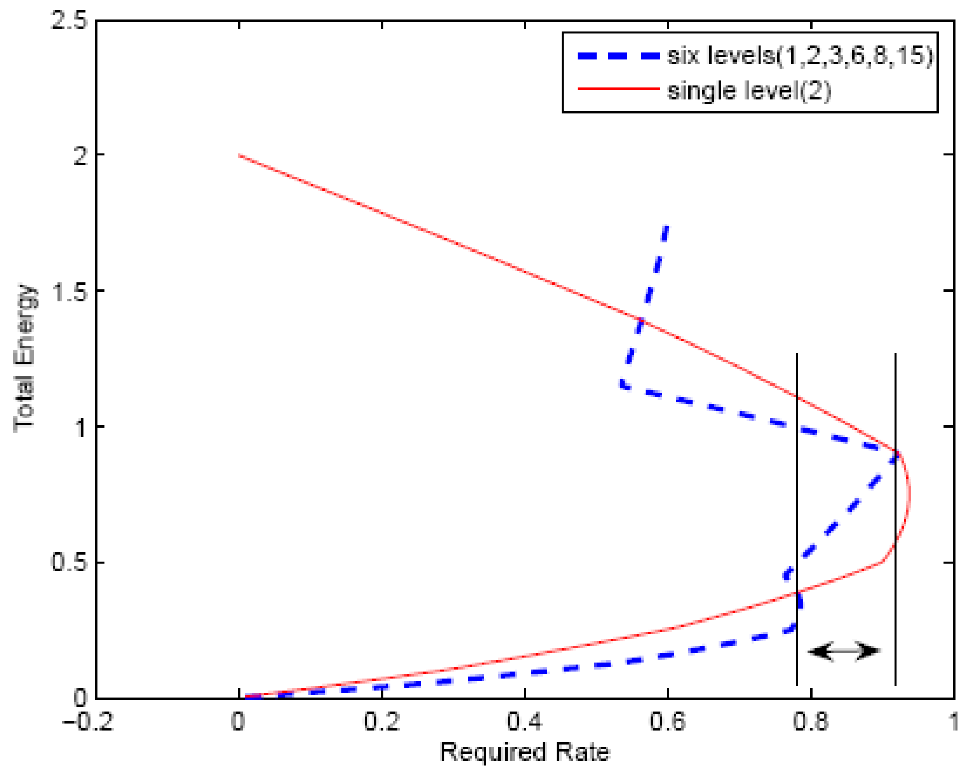
- There always exists a uniformly best equilibrium (or none).
- The best-response with “slow start” converges to the best equilibrium.

But:

- Multiple (more than 2) equilibrium points may exist.
- The best equilibrium need not be socially optimal.
- A Braess-like paradox may exist: addition of possible power levels to some (or all) users might lead to decreased channel capacity, and worse performance for all.

Illustrative example:

- Two symmetric users, with one /multiple power levels:



Conclusion

- We introduced and studied the concept of fix-rate equilibrium in wireless collision channels within a distributed / non-cooperative framework, with and without channel fading and power level control.
- Many issues remain for further study. Among those we mention:
  - Incorporating channel reservations (RTS/CTS)
  - Detection and avoidance of worse equilibria
  - Correlated channel fading across users
  - Extending the user objective: elastic demand, rate regulation
  - Non-stationary transmission strategies
  - More general (capture) channels [INFOCOM 2008]

Thank You

☞ GameComm 2008: One-day Workshop on Game Theory in Communication Networks, Athens, October 20. Paper submission: June 10.