Exam (Moed Aleph) in Distributed Algorithms

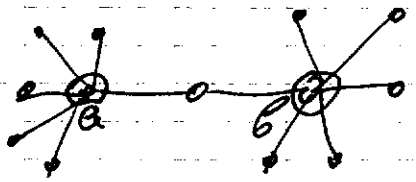
Solve 3 questions out of four.

1. Def: A vx set $U \subseteq V$ is called a k-ruling set, for a pos' int' param' k, if

(1) for every vx $v \in V$ there exists a "ruling" vx $u \in U$ with $dist_G(u,v) \leq k$.

(2) Every two vs $u, u' \in U$ are at distance at least 2 one from another.

Example: The set $\{a, b\}$ is a 1-ruling set for this graph.

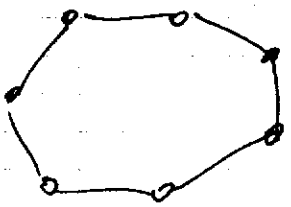


The problem: We are given an L-coloring, for a positive integer L. Devise an algorithm that builds an $O(\log L)$-ruling set for a given graph G (for which we have an L-coloring). Analyze its running time and prove its correctness.

2. Def: A coloring φ of a graph $G=(V,E)$ is called an $\mathcal{L}$-coloring $\mathcal{B}$-defective (for a pair of int' param's $\mathcal{L}$ and $\mathcal{B}$) if φ assigns every vx a color from $[\mathcal{L}]$ and for every vx $v \in V$, it has at most $\mathcal{B}$ neighbors of the same color.

Problem: Given an $\mathcal{L}$-coloring $\mathcal{B}$-defective φ of a graph $G=(V,E)$ w/ $\Delta(G)=\Delta$ and $\sqrt{\Delta} \leq \mathcal{L}$, $\mathcal{B} \leq \sqrt{\Delta} + O(1)$. Devise an efficient $(\Delta-1)$-coloring alg' (that might employ this defective coloring).

3) Invoke Mycielski's construction on C_7. Describe the resulting graph. What are its properties?

C_7 

(What is its girth, chromatic number, and number of vs?)

4) Define the notion of k-spanner. Describe the notion of δ synchronizer based on k-spanner, for a fixed k. Prove its correctness.

What are $Time_{pulse}(\delta)$, $Comm_{pulse}(\delta)$? How do these compare with $Time_{pulse}(\delta')$, $Comm_{pulse}(\delta^{\infty})$, and with $Time_{pulse}(\mathcal{L})$, $Comm_{pulse}(\mathcal{L})$, and with $Time_{pulse}(\mathcal{B})$, $Comm_{pulse}(\mathcal{B})$?

Good luck!