ASSIGNMENT 4 - DISTRIBUTED ALGORITHMS

DUE DATE - 29/1/2007

(1) Given a rooted tree (T, rt), in which each vertex knows the identities of its parent, its children and itself. Describe a distributed algorithm for minimal coloring of this tree that requires exactly 7 time steps. Analyze its time and communication complexity and prove its correctness.

(2)

Definition 0.1. Given a graph $G = (V, E)$, a group $U \subseteq V$ is a *Maximal Independent Set* (*MIS*) if:

- for each pair of vertices $u, w \in U$: $(u, w) \notin E$
- for each vertex $v \notin U$ there exist a vertex $u \in U$ so that $(u, v) \in E$

Given a distributed 3-coloring algorithm Π that works on rooted trees (T, rt) and a *MIS* U of T; each vertex $v \in V(T)$ knows whether $v \in U$ or not. The algorithm Π might use U.

Let $Time_n(\Pi)$ be the worst case runtime of Π on a tree T in which $|V(T)| = n$. Describe a most efficient (in terms on time) distributed algorithm Π' that finds a 3-coloring of a path $p, |p| = n$. Π' can use the given algorithm Π, but note that Π needs a *MIS* as an input. Prove your algorithm correctness, and analyze its runtime and communication complexity (you can use $Time_n\Pi$ for this purpose).

(3) Given a circular road with n gas pumps. Each pump holds a given amount of gas, so that the total amount of gas in all pumps is the exact amount required for a car to complete a lap on the road.

(a) Prove or disprove:

Lemma 0.2. *For each such road, there exist a gas pump in which a car can be placed so it would be able to complete a lap on the road without getting out of gas. The car starts with no gas at all.*

(b) If the lemma is correct, describe a synchronous distributed algorithm that finds that gas pump. Assume that each gas pump is a processor, and two neighbour pumps can communicate by sending messages. Prove its correctness and analyze its runtime and communication complexity.

(4) Suggest a most efficient distributed algorithm that finds a 3-coloring on a ring. Prove its correctness and analyze its runtime and communication compexity.

(5) Let η be a synchronizer, and $G = (V, E)$ a graph.

Definition 0.3. $t_{max}(x) = max\{p_v(x) | v \in V\}$, where x is a time step and $p_v(x)$ is the value of the pulse variable of the vertex v.

Definition 0.4. $t_{min}(x) = min\{p_v(x) | v \in V\}$

Let $s_{max} = sup\{t_{max}(x) - t_{min}(x) | x \in \Re^{+}$ [1] $\}$ $(s_{max} \in \Re^{+})$[2].
Prove that: $Time_{gap}(\eta) \leq (s_{max} + 1) \cdot Time_{pulse}(\eta)$.

[1] $\Re^{+}$ is the group of non-negative real numbers
[2] t_{max} and t_{min} are taken on all the possible runs of η for each distributed algorithm Π