Assignment 1 - Distributed Algorithms

Due Date - 26/11/2006

(1) (a) Prove that:

$$Rad(G, v) \leq Diam(G) \leq 2 \cdot Rad(G, v)$$

for each graph $G = (E, V)$ and vertex $v \in V$.

(b) Provide an example of a graph $G = (E, V)$ and a vertex $v \in V$ in which $Diam(G) < 2 \cdot Rad(G, v)$.

(2) (a) Prove that a connected graph with n vertices contains at least $n - 1$ edges.

(b) Prove that a connected graph with n vertices and $n - 1$ edges is necessarily a tree.

(3) Denote $\check{L}(v)$:

$$\check{L}(v) = \begin{cases} 0, & \text{v is a leaf} \\ 1 + min\{\check{L}(u) : u \in Ch(v)\}, & \text{otherwise} \end{cases}$$

Prove that $\check{L}(v)$ is the minimum distance between v to any leaf in the subtree of T rooted at v.

(4) Let $Prune(T)$ be a procedure that deletes an arbitrary leaf, chosen randomally by the procedure, from the rooted tree (T, rt). Let (T', rt) be the rooted tree which is the result of applying several times $Prune(T)$. (The number of times $Prune(T)$ is applied is not known; We assume that v $\in$ T' , i.e that v was never deleted). Prove or disprove the following claims:

(a) $L_{T'}(v) \leq L_T(V)$ (recall that $L_T(v) = dist_T(v, rt)$)

(b) $\hat{L}_{T'}(v) \leq \hat{L}_T(v)$ (recall that $\hat{L}_T(v) = Depth(T, v)$ - i.e , the depth of the subtree of T rooted at v)

(c) $\check{L}_{T'}(v) \leq \check{L}_T(v)$

(5) (a) Describe a distributed algorithm in the asynchronous model that counts the number of vertices in a graph. The algorithm should be activated by some vertex, denoted by x, and at the end of the execution of the algorithm , vertex x should know the number of vertices in the graph.

(b) Extend the algorithm from previous section, such that at the end of the execution of the algorithm all vertices would know the number of vertices in the graph.

Grader's Note: 10 pt. bonus will be given for printed works!