ASSIGNMENT 2 - DISTRIBUTED ALGORITHMS

Due Date - 19/12/05

(1) (Upcast with unordered items): Given a rooted tree (T, rt) and m items $P_1, .., P_m$ which are initially stored at some of the vertices of the tree T in an arbitrary manner. Items can be replicated, namely, each item is stored in one or more vertices (and each vertex may store zero or more items). The goal is to end up with all the items stored at the root of the tree, rt. The items are entirely incomparable (namely, for any two items stored at some vertex, that vertex only knows that these items are distinct, but does not know their indices and cannot determine whether the index of one item is greater than the index of the other item).
   (a) Describe a distributed algorithm for solving the problem in the synchronous model. Analyze the time and message complexities of your algorithm.
   (b) Same as (a) but in the asynchronous model.

(2) Given a rooted tree (T, rt) and a set of $2k$ vertices $W = \{w_1, ..., w_{2k}\}, W \subseteq V(T)$, and each vertex knows whether it belongs to W or not.

Definition 0.1. Let f be a bijection from W to W. For a vertex $w \in W$, denote the unique path between w and $f(w)$ in the tree T by $P_w(f)$.

   (a) Prove that there exists a bijection $f : W \to W$, f is not the identity mapping, such that for all pairs of distinct vertices $w, w' \in W$, such that $w' \neq f(w)$, the paths $P_w(f)$ and $P_{w'}(f)$ are edge-disjoint.
   (b) Describe a distributed algorithm that finds the mapping of f in the synchronous model. At the end of the execution of the algorithm, each vertex w in W should know the identifier of the vertex with which it is paired, namely the value of $f(w)$, and which of its incident edges is in $P_w(f)$. Analyze the time and message complexities of your algorithm.