CHARACTERISTIC FUNCTIONS AND TRANSFER FUNCTIONS IN OPERATOR THEORY AND SYSTEM THEORY:
A conference dedicated to Paul Fuhrmann on his 70th anniversary and to the memory of Moshe Livsic on his 90th anniversary.

Organizers: D. Alpay and V. Vinnikov

Ben-Gurion University of the Negev, Beer-Sheva
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ABSTRACTS

Passive realizations of passive discrete time behaviors

Damir Arov and Olof Staffans

We present and study several different types of passive state/signal realizations of passive discrete time behaviors, such as simple conservative, minimal passive, minimal optimal, minimal $*$-optimal, and balanced passive state/signal realizations. The Krein space geometry plays a central role in this study, since the interchange of energy between the system and its surroundings is modeled in terms of Krein signal space, and since the node space contains both a positive copy (representing past time) and a negative copy (representing future time) of the state space. In this lecture we introduce the notion of inner scattering channels and the corresponding Hankel operator of a simple convervative state/signal system, and we present criteria for similarity or unitary similarity of all minimal passive state/signal realizations of a given passive behavior.

The characteristic function as a unitary invariant: Hilbert space contraction operators to completely contractive representations of exotic disk algebras

Joe Ball

The characteristic function originated from various points of view in work of Livšic, Sz.-Nagy-Foias and de Branges-Rovnyak from varying points of view as unitary invariant for a Hilbert space contraction operator. With some additional hypotheses the characteristic function is more tractable than the original operator and serves as a function-theoretic tool for understanding the structure of a nonunitary operator. Recently there have been extensions of the notion of characteristic function as a unitary invariant for more general kinds of operator-theoretic objects, e.g.: a time-varying contraction operator, a commutative row contraction, a freely noncommutative row contraction, a contractive row contraction with components satisfying a given set of relations, a completely contractive representation of a nest algebra, or, more generally, a completely contractive representation of a path algebra or quiver algebra. A general setting encompassing all of the above is the characteristic function for a completely contractive representation of the generalized H^∞-algebra associated with the Fock space of a W^*-correspondence introduced and studied by Muhly and Solel. Results are definitive in some of these settings but have an incomplete character in the most general case. The talk will give an overview of these developments.

Fuhrmann's realization theory and tangential interpolation in rational approximation

Laurent Baratchart

Abstract: we discuss the tangential interpolation equations arising in L2-best rational approximation of fixed Mc-Millan degree to a given matrix-valued transfer-function F. We show how Fuhrmann's realization theory allows one to obtain a matrix analog of Walsh's theorem, and to further analyse the rank of the error function. In particular, we show if F is rational of degree k and H is an interpolant of degree n that F-H has rank at most k-n. This entails a consistency property in minimum variance identification by stable rational models (whose poles are not restricted to lie in a compact subset of the disk).

Operator models for Nevanlinna families in reproducing kernel Hilbert spaces

Jussi Behrndt

The class of Nevanlinna families consists of R-symmetric holomorphic relation functions on $\mathbb{C}\setminus\mathbb{R}$ with maximal dissipative (maximal accumulative) values on $\mathbb{C}_+$ ($\mathbb{C}_-$, respectively) and extends the class of operator-valued Nevanlinna functions. In this talk it will be shown that every Nevanlinna family can be realized as the Weyl family of a boundary relation induced by the multiplication operator with the free variable in a reproducing kernel Hilbert space. This can be regarded as a natural generalization of the well known fact that each operator-valued Nevanlinna function with boundedly invertible imaginary part can be realized as a Q-function or Weyl function of a symmetric and a selfadjoint multiplication operator. Our construction is based on a modification of the de Branges-Rovnyak model for the realization of Schur functions as transfer functions of unitary colligations.

This talk is based on joint work with Henk de Snoo and Seppo Hassi.

On abstract interpolation problem in Nevanlinna classes

Vladimir Derkach

A reproducing kernel Hilbert space model for a Nevanlinna pair is used in order to formulate a continuous version of the abstract interpolation problem that was posed originally by V. Katsnelson, A. Kheifets and P. Yuditskii for the Schur class. A description of the set of solutions of this problem is obtained via M. G. Krein's representation theory of symmetric operators. A general approach to degenerate interpolation problems is discussed. The results are illustrated with the example of a degenerate truncated moment problem.

On the Numerical Hankel Rank of Schur complements of Discretized Elliptic PDE's in 2D

Patrick Dewilde

The mathematical properties of Schur complements play an important role in solving discretized systems of PDE's or in computing eigenvalues. When a lexicographic ordering of entries is used, then the Schur complements satisfy an algebraic Riccati equation, whose solution can be determined explicitly. We show that this solution has special properties, namely that its so called off-blocks have low numerical Hankel rank, meaning that they can be approximated or model reduced by a low complexity semi-separable system. The theory has important applications in signal processing and in efficient solvers of partial differential equations.

Joint work with S. Chandrasekharan (UCSB), and M. Gu (UCB).

Augmented Schur parameters for generalized Nevanlinna functions and approximation

Aad Dijksma

Schur parameters of a Schur function are a well known concept in Schur analysis. It ha recently been generalized to generalized Nevanlinna functions. We define and study the analog of this extended concept, and apply it to approximation of generalized Nevanlinna functions by rational functions.

Joint work with D. Alpay and H. Langer.

Applications of Realizations

Michael Dritschel

Some recent versions of realization theorems will be presented and a handful of the many applications outlined. The talk is based mostly on joint work with Scott McCullough, as well as Stefania Marcantognini, Milne Anderson, Jim Rovnyak and Tirtha Bhattacharyya.

Cascade representation and minimal factorization of multidimensional systems.

Chen Dubi

Consider a rational matrix valued function $R(z)$in N complex variables with the following realization:

$$R(z) = D + C(I - \sum_{j=1}^{N} z_j A_j)^{-1} \sum_{j=1}^{N} z_j B_j.$$

It is well known that *every* matrix valued function can be realized in such a manner. The last realization can be identified with the transfer function of a Fornasini-Marchesini Linear system, defined by:

$$\begin{aligned} X_\alpha &= \sum_{j=1}^{N} A_j X_{\alpha - e_j} + \sum_{j=1}^{N} B_j U_{\alpha - e_j} \\ Y_\alpha &= C X_\alpha + D U_\alpha \end{aligned}$$

As turns out, there is a strong correlation between factorization of the transfer function and cascading of the linear system.
In the talk we will discuss the relation between the two notions, and present necessary sufficient conditions for minimal factorization of a transfer function on one hand, and for a simple cascade representation of a Fornasini-Marchesini system on the other.
Doing so, we will generalize the well known criterion of Bart, Gohberg, Kaashoek and Van Dooren for minimal factorization of rational functions in a single variable.
The presented study is a joint work with Daniel Alpay.

Degree bounds on polynomials of symmetric matrices,

Harry Dym

A symmetric polynomial $p(X) = p(X_1, \ldots, X_g) = p(X_1, \ldots, X_g)^T$ of real symmetric $n \times n$ matrices $X_1, \ldots, X_g$ with real coefficients is said to be matrix convex if

$$p(tX + (1-t)Y) \preceq tp(X) + (1-t)p(Y)$$

for every pair $(X_1, \ldots, X_g)$ and $(Y_1, \ldots, Y_g)$ of tuples of real symmetric $n \times n$ matrices and every choice of t in the interval $0 \leq t \leq 1$. A theorem of Helton-McCullough states that if p is matrix convex, then p is a polynomial of degree less than or equal two.
In this talk I shall discuss bounds on the degree d of p in terms of the signature of an associated block matrix $\mathcal{Z}$ with entries $\mathcal{Z}_{ij} \in \mathbb{R}^{g^{i+1} \times g^{j+1}}$, $i, j = 0, \ldots, d-1$, which, as a byproduct, yields a new proof of the theorem of Helton-McCullough referred to above.
The talk is based on joint work with J. William Helton and Scott McCullough. It will be expository.

REFERENCES

Harry Dym, J. William Helton and Scott McCullough, *The Hessian of a noncommutative polynomial has numerous negative eigenvalues*, J. d'Analyse, in press.

On Operator Algebras Determined by a Sequence of Operator Norms

Avraham Feintuch

We consider a family of operators determined by a sequence of operator norms. When the sequence of norms is determined by a single operator the natural question that arises is when the algebra properly contains the commutant of the operator. In this case the existence of invariant subspaces for the algebra is stronger than the existence of hyperinvariant subspaces for the operator.

This is joint work with Alexander Markus.

Constrained interpolation: loopshaping and the Kimura-Georgiou parametization,

Andrea Gombani

(joint work with Gy. Michaletzky) We show here how a general characterization of fixed degree interpolating functions in terms of Sylvester equations yields a parametrization of the so called "loopshaping" functions, i.e. all functions which are analytic in a given domain, meet some interpolating conditions there and satisfy some non constant constraints on the boundary. We also show how this characterization connects with the Kimura-Georgiou parametrization.

The inverse problem for Kreĭn orthogonal matrix functions

M.A. Kaashoek

Let $k \in L_1^{n\times n}[-\omega,\omega]$, and let k be hermitian, that is, $k(t)^* = k(-t)$ for $-\omega \le t \le \omega$. Assume the equation

$$\varphi(t) - \int_0^\omega k(t-s)\varphi(s)\,ds = k(t), \quad 0 \le t \le \omega, \tag{1}$$

has a solution $\varphi \in L_1^{n\times n}[0,\omega]$, and consider the associated entire $n\times n$ matrix function

$$\Phi(\lambda) = I + \int_0^\omega e^{i\lambda t}\varphi(t)\,dt, \quad \lambda \in \mathbb{C}. \tag{2}$$

For the scalar case ($n = 1$) functions of type (2) determined by (1) have been introduced by M.G. Kreĭn as continuous analogues of the classical Szegö orthogonal polynomials with respect to the unit circle. For this reason Φ is referred to as a *Kreĭn orthogonal matrix function* generated by k.
In this talk we deal with the associated inverse problem. Necessary and sufficient conditions are presented for a function Φ defined by (2) to be a Kreĭn orthogonal matrix function. The main result is a generalization to the matrix case of the corresponding Kreĭn-Langer ('85) result for scalar functions. The analysis is more involved than the one for the scalar counterpart. Solutions to certain equations in entire matrix functions enter into the proof. Recent results from the theory of continuous analogues of the resultant are also used. The talk is based on joint work with I. Gohberg and L. Lerer.

Conservative dilations of dissipative multidimensional systems in the commutative and non-commutative settings

Dmitry Kalyuzhnyi-Verbovetzkii

A version of the celebrated Sz.-Nagy Dilation Theorem for 1D linear time-invariant systems which is known since the work of Arov in 1979 is generalized to various classes of non-commutative nD systems. As a consequence of this result, the criteria for existence of conservative dilations are obtained for the commutative nD counterparts of these systems. The talk is based on the joint work with J. A. Ball.

Toeplitz C^*-Algebras on Dirichlet Spaces of the Ball

H. Turgay Kaptanoğlu

Dirichlet spaces $\mathcal{D}_q$ ($q \in \mathbb{R}$) are Hilbert spaces of holomorphic functions on the unit ball of $\mathbb{C}^N$ defined by a reproducing kernel which is $K_q(z,w) = (1-\langle z,w\rangle)^{-(N+1+q)}$ for $q > -(N+1)$ and given by a hypergeometric function for $q \leq -(N+1)$.
We consider the C^*-algebra $\mathcal{T}_q$ generated by the N-tuple of operators of multiplication by the coordinate functions (the so-called N-shift) on $\mathcal{D}_q$. We show that $\mathcal{T}_q$ contains all compact operators and Toeplitz operators with continuous symbols, a quotient map sends it onto the continuous functions on the boundary of the unit ball, and thus obtain the related short exact sequence of C^*-algebras. This result generalizes what is known for the Hardy space ($q = -1$), Bergman spaces ($q > -1$), and the Drury-Arveson space ($q = -N$).
We also show that the symmetric Fock space over $\mathbb{C}^N$ can be realized as each of the Dirichlet spaces under a suitable norm as noticed earlier for the Arveson space ($q = -N$).

Joint system realizations, rational solutions of the Schlesinger system and related tau functions

V. Katsnelson

Origins of the system realization theory may be found in many different fields: for instance, scattering problems in physics, synthesis problems in electrical networks and control problems in linear dynamical systems. For many mathematicians the story begins with the theory of operator colligations and their characteristic functions which was introduced by M. Livšic. Investigations of M. Livšic were continued by L. Sakhnovich, who considered, in particular, the spectral factorization and classical interpolation of a rational matrix function, when both the function and its inverse are given as transfer functions of linear systems, and discovered the key role of matrix equations of the form

$$AS - SB = C$$

(a.k.a. the Sylvester-Lyapunov equations) in these problems. Later system realizations of rational matrix functions and their inverses were given a complete treatment in a series of works by J. Ball, I. Gohberg, M. Kaashoek, L. Rodman *et. al.* on reconstruction of rational matrix functions from the local data - zeros and poles.
It turns out that the above mentioned results are very useful in the study of holomorphic families of rational matrix functions parameterized by the position of zeros and poles, which possess various iso- (from $\bar{\iota}\sigma o\varsigma$ - equal - in Old Greek) properties. In particular, one can construct explicitly rational solutions of the Schlesinger system of partial differential equations. For these solutions the determinant of the matrix S, which satisfies the appropriate Sylvester-Lyapunov equation, coincides with the tau function introduced by M. Sato, T. Miwa and M. Jimbo. The theory also allows to construct explicit counterexamples to the theorems of L. Schlesinger and T. Miwa - that is, isomonodromic families which are *not* solutions of the Schlesinger system and are *not* meromorphic with respect to the parameter.

This is a joint work with D. Volok.

Direct and Inverse Lifting Problem

Alexander Kheifets

A straightforward parametrization of all contractive lifts of a contractive intertwiner is given based on the dynamical approach and on "time" versions of the scattering function and the Fourier representation. A characterization of the "coefficients" in the parametrization formula is obtained (necessary and sufficient properties are proved, including analogues of Adamjan-Arov-Krein theorem and factoral optimality of the "coefficients"). Then the results are translated into the "functional" context. Based on a joint work with Joe Ball.

Spectral points of definite type and the Virozub - Matsaev condition

Heinz Langer

For a self-adjoint operator A in a Krein space $(\mathcal{K}, [\cdot,\cdot])$ or for a self-adjoint operator function T in a Hilbert space $(\mathcal{H}, (\cdot,\cdot))$, the real point $\lambda \in \sigma(A)$ or $\in \sigma(T)$ is said to be *of positive type* if

$$x_n \in \mathcal{K},\ \|x_n\| = 1,\ \lim_{n\to\infty}(A-\lambda)x_n = 0 \Longrightarrow \liminf_{n\to\infty}\,[x_n, x_n] > 0$$

or

$$x_n \in \mathcal{H},\ \|x_n\| = 1,\ \lim_{n\to\infty} T(\lambda)x_n = 0 \Longrightarrow \liminf_{n\to\infty}\,(T'(\lambda)x_n, x_n) > 0.$$

On an interval I which contains spectral points only of positive or only of negative type for A or T, there exists a spectral function. Consequently, the operator A or the linearization of the operator function T have the same spectral behavior on this interval I as a self-adjoint operator in Hilbert space. For an operator function T, a sufficient condition for spectrum of positive type is the Virozub-Matsaev condition (VM):

$$x_n \in \mathcal{H},\ \|x_n\| = 1,\ \lim_{n\to\infty}\,(T(\lambda)x_n, x_n) = 0 \Longrightarrow \liminf_{n\to\infty}\,(T'(\lambda)x_n, x_n) > 0.$$

We formulate simpler sufficient conditions for (VM) and discuss related questions.
Joint work with M. Langer, A. Markus, C. Tretter .

On Large Deviations Laws for Consistent Parameter Estimates in Diffusions,

David Levanony

Rates of convergence of consistent parameter estimates in diffusion processes are studied via large deviations (LD) laws for the suprema of the estimation error's tail processes. First,we derive conditional LD laws for properly normalized martingale processes satisfying a martingale Law of Large Numbers (LLN). These are then utilized to obtain corresponding laws for the estimation error's tail processes. Resulting simple stopping rules are presented. Finally, unconditional LD lower bounds are discussed.

Convex Invertible Cones and Partial Ordering in Control Theory

Izchak Lewkowicz

Let B be an arbitrary matrix within the Convex Invertible Cone generated by A (which has no purely imaginary eigenvalues). We show that this amounts to a partial ordering in the context of Control Theory:
- B may be obtained from A as a solution to a Nevanlinna-Pick interpolation problem.
- The set of all Lyapunov factors associated with A is contained in those associated with B.
- For an arbitrary vector b, the controllable subspace of B, b is contained in that of A, b.
- If A is Hurwitz stable, the "variance" of all solutions of the differential equation $\frac{dx}{dt} = Bx$ is smaller than that of $\frac{dx}{dt} = Ax$.

Joint work with Nir Cohen.

On convexity of ranges of quadratic forms on various sets

Alexander Markus

This is a joint work with I. Feldman and N. Krupnik.
Let H be a complex inner product space. By the famous Töplitz - Hausdorff theorem, the range of any quadratic form on the unit sphere S of the space H is convex. We consider the following question: which subsets of H besides S have this property (*Töplitz -Hausdorff property*)? One of the obtained results follows.
Theorem. *Let* $H = \boldsymbol{C}^n$, $0 < p < \infty$ *and* $0 < r \leq R < \infty$. *The set*

$$\{z \in \boldsymbol{C}^n : r^p \leq \sum_{k=1}^{n} |z_k|^p \leq R^p\}$$

has the Töplitz - Hausdorff property if and only if $p = 2$.
Corollary. *The unit sphere in* l^p*-norm has the Töplitz - Hausdorff property if and only if* $p = 2$.

State-space approach to zero-modules of proper transfer functions,

Gyorgy Michaletzky

The poles and zeros of a transfer function can be studied by various means. The main purpose of the present paper is to give a state-space description of the module theoretic definition of multivariate zeros defined and analysed by Wyman et al. This analysis is carried out for proper transfer functions. Not surprisingly some notions from geometric system theory play important role in this analysis. The output-nulling controlled-invariant subspace, the set of output-nulling reachable elements and the output-nulling reachability subspace determine a decomposition of the state-space reflecting the various zero-modules: finite zeros, infinite zeros, and the Wedderburn–Forney spaces based on the kernel and the image subspace of the transfer function (leading to the so-called generic zeros). The notion generic refers to the fact that the exact position of these zeros are not specified, they occur everywhere. But - under some conditions - multiplying with appropriate (uniquely defined) inner-functions from the left and from the right they can be turned into finite zeros (with uniquely defined positions). I. e. using the terminology introduced by P. Furhmann and A. Gombani the externalized zeros can be internalized. The proof utilizes the fact that the corresponding subspaces are orthogonal to each other when the same proper rational function is considered as a transfer function for the left vs. right multiplication.

The Curvature invariant for a class of homogeneous operators

Gadadhar Misra

For an operator T in the class $\mathrm{B}_n(\Omega)$, introduced by Cowen and Douglas, the simultaneous unitary equivalence class of the curvature and the covariant derivatives up to a certain order of the corresponding bundle E_T determine the unitary equivalence class of the operator T. In a subsequent paper, the authors ask if the simultaneous unitary equivalence class of the curvature and these covariant derivatives are necessary to determine the unitary equivalence class of the operator $T \in \mathrm{B}_n(\Omega)$. Here we show that some of these covariant derivatives are necessary. Our examples consist of homogeneous operators. For homogeneous operators, the simultaneous unitary equivalence class of the curvature and all its covariant derivatives are determined from the simultaneous unitary equivalence class of these at 0. This shows that it is enough to calculate all the invariants and compare them at just one point, say 0. These calculations are then carried out in number of examples. One of our main results is that the curvature along with its covariant derivative of order $(0, 1)$ at 0 determines the equivalence class of generic homogeneous holomorphic Hermitian vector bundles over the unit disc.

This is joint work with Subrat Shyam Roy.

On the spectral analysis of nonself-adjoint operators with almost Hermitian spectrum,

Sergey Naboko

Nonself-adjoint,nondissipative operators with real pure singular spectrum are considered.Using a functional model based on the Livsic Characteristic Function as a principal tool,spectral properties of such operators are investigated.In particular a version of the Cayley Identity is one of the applications.

Perturbation analysis of Lagrangian invariant subspaces of symplectic matrices

Leiba Rodman

A subspace of a finite dimensional real or complex vector space is said to be Lagrangian (relative to a given nondegenerate bilinear or sesquilinear form) if it is neutral with respect to the form and the dimension of the subspace is equal to half the dimension of the vector space. Lagrangian invariant subspaces for symplectic matrices play an important role in the numerical solution of discrete time, robust and optimal control problems. The sensitivity (perturbation) analysis of these subspaces, however, is a difficult problem, in particular, when the eigenvalues are on or close to some critical regions in the complex plane, such as the unit circle.
A detailed perturbation analysis is presented for several different cases of real and complex symplectic matrices. Stability and conditional stability of Lagrangian invariant subspaces of these matrices are analyzed, as well as the index of stability for these subspaces.
(Join work with C. Mehl, V. Mehrmann, and A. C. M. Ran.)

Integral Equations in the Theory of Levy Processes

Lev Sakhnovich

We consider the Levy processes and the corresponding semigroups. The generators of these semigroups we represent in convolution forms. Such a representation opens a possibility to use the theory of integral equations with difference kernels. The probability of Levy process remaining within the given domain (ruin problem) is investigated. M. Kac had obtained the first results of this type for the symmetric stable processes. (see H.Widom .) We compare the obtained results with the well-known results: the iterated logarithm law, the results for the first hitting time, the results for the most visited sites. We analyse in detail a number of concrete examples of the Levy processes which are used in the financial mathematics: stable processes, Wiener processes, Gaussian process, Meixsner process. We use the following methods: M.Livshits theory, method of operator identities, M . Kac approach, Krein-Rutman theorem.

The Beginning,

Mikhail Sodin

I will discuss the very first publication (1939) of Moshe Livsic where he studied the entries of J-contracting entire matrix functions that appear in the parameterization of solutions of the indeterminate Hamburger's moment problem.
Surprisingly, 85 years after the works of Hamburger, M. Riesz and R. Nevanlinna, and almost 70 years after the work of Livsic, we do not know the answers to basic intrinsic questions concerning these matrices.

Operator Algebras associated with Unitary Commutation Relations

Baruch Solel

This is a joint work with S. Power. We study nonselfadjoint operator algebras with two sets of generators subject to a unitary commutation relation between the two sets of generators. This relation is given by a unitary matrix u and we classify the algebras (up to isometric isomorphism) in terms of the matrix u.

Dirichlet eigenvalues in a narrow strip

Michael Solomyak

(joint work with Lennie Friedlander, Arizona)

We study the Dirichlet Laplacian Δ_ε in the domain

$$\Omega_\varepsilon := \{-a < x < b, \qquad 0 < y < \varepsilon h(x)\}.$$

Here $h(x)$ is a smooth function on $[-a, b]$, with $x = 0$ as the only point of global maximum, say M. We admit that

$$h(x) = M - cx^{2m} + O(x^{2m+1}) \qquad \text{as } x \to 0,$$

with $M, c > 0$ and $m \in N$.
PROBLEM:
WHAT IS THE BEHAVIOUR OF THE EIGENVALUES $\lambda_j(\varepsilon)$ as $\varepsilon \to 0$?
ANSWER: TWO-TERM ASYMPTOTICS. Consider the operator on $L^2(\mathbb{R})$

$$\mathbf{H} = -\frac{d^2}{dx^2} + qx^{2m}, \qquad q = 2\pi^2 cM^{-3}.$$

Then

$$\lambda_j(\varepsilon) \sim \frac{\pi^2}{M^2\varepsilon^2} + \frac{\lambda_J(\mathbf{H})}{\varepsilon^{2\alpha}}, \qquad \alpha = (m+1)^{-1}.$$

We also show that convergence of eigenvalues here is a consequence of some ‘generalized’ version of the convergence in norm of the resolvents. A modificaton of the standard resolvent convergence is necessary, since the operators Δ_e for different ε, as well as the operator $\mathbf{H}$, act in different spaces.

Carathéodory–Herglotz functions in the Banach space case,

Olga Timoshenko

Let $\mathcal{B}$ be a Banach space. An $\mathbf{L}(\mathcal{B},\mathcal{B}^*)$-valued function Φ defined in the open unit disk $\mathbb{D}$ (but *a priori* without any analyticity property) is called a Carathéodory-Herglotz function if the kernel

$$K_\Phi(z,w) = \frac{\Phi(z) + \Phi(w)^*\big|_{\mathcal{B}}}{1 - z\overline{w}}$$

is positive in $\mathbb{D}$. We denote by $\mathcal{C}(\mathcal{B},\mathcal{B}^*)$ the family of $\mathbf{L}(\mathcal{B},\mathcal{B}^*)$-valued Carathéodory-Herglotz functions. In the case when $\mathcal{V}$ is a Hilbert space $\mathcal{H}$ we use the notation $\mathcal{C}(\mathcal{H})$.

Theorem: $\Phi \in \mathcal{C}(\mathcal{B},\mathcal{B}^*)$ *if and only if*

$$\Phi(z) = D + T^*\varphi(z)T,$$

where $D \in \mathbf{L}(\mathcal{B},\mathcal{B}^*)$ *is purely imaginary,* $T \in \mathbf{L}(\mathcal{B},\mathcal{H})$ *for some Hilbert space* $\mathcal{H}$ *and* $\overline{\text{ran}\ (T)} = \mathcal{H}$, $\varphi \in \mathcal{C}(\mathcal{H})$. *Moreover, given* Φ, *the operator* D *is determined uniquely while the operator* T *and the space* $\mathcal{H}$ *are determined up to unitary mappings.*

We can reduce interpolation problems for which the value of the function $\Phi \in \mathcal{C}(\mathcal{B},\mathcal{B}^*)$ is pre-assigned at the origin to interpolation problems for functions $\varphi \in \mathcal{C}(\mathcal{H})$.

We also consider the case where one replaces $\mathcal{B}$ by a locally convex topological space which admits the factorization property.

Joint work with D. Alpay and D. Volok.

Differential-difference equations in the space of entier functions,

Vadim Tkachenko

(joint work with Genric Belitski)

We consider a difference-differential equation

$$\sum_{k=1}^{m}\sum_{j=0}^{q_k} a_{kj}(z)\phi^{(j)}(z+\tau_k)=\gamma(z) \tag{$*$}$$

where $\tau_1 < ... < \tau_m$ are distinct real numbers, and $a_{kj}(z)$ and $\gamma(z)$ are entire functions. We assume that
1. The functions $a_{1q_1}(z)$ and $a_{mq_m}(z)$, i.e., the leading coefficients of the extreme differential operators do not vanish anywhere in the complex plane;
2. There exists a positive increasing function $w(t)$ satisfying

$$w(t+s)\le w(t)+w(s), \qquad \int_1^\infty \frac{\ln w(t)}{t^2}<\infty,$$

and such that for every $h>0$ there exists a number $C(h)$ for which the estimates

$$\left|\frac{a_{kj}(z)}{a_{1q_k}(z)}\right| \le C(h)e^{w(|\mathrm{Re}z|)}, \quad k=1,m; \quad j=0,1,...,q_k;$$

$$ln\left|\frac{a_{pj}(z)}{a_{1q_k}(z)}\right| \le C(h)e^{w(|\mathrm{Re}z|)}, \quad k=1,m; \quad 1<p<m; \quad j=0,1,...,q_p;$$

hold in the strip $\{z : |\mathrm{Im}z| \le h\}$.
With the above conditions being satisfied we prove the following propositions:
1. For every entire function $\gamma(z)$ Equation (*) has an entire solution $\phi(z)$.
2. The space of entire solutions of homogenuous Equation (*) is infinite-dimensional.

An internal model principle for observers

Jochen Trumpf

An observer is a device which takes a set of measured signals as input and produces an estimate of another set of signals as output. Here, it is assumed that the two sets of signals are interacting dynamically, and that the dynamical system describing their interaction, the observed system, is known. The interconnection of the observed system with the observer will give rise to a set of error signals, namely the differences between the to be estimated signals and the actual estimates.

We consider the case of finite dimensional linear, time-invariant, differential (LTID) observed systems and observers. Then, by the elimination theorem, the set of error signals forms a finite dimensional LTID system as well, the error system. Usually, we want the error system to be stable, so that we are guaranteed to get an at least asymptotically accurate estimate. The corresponding observers are usually called asymptotic observers or stable observers. We prove that in the above setting any observer that leads to an autonomous error system (hence, in particular any asymptotic observer) will contain an internal model of the controllable part of the observed system. This result generalizes a theorem proved in 2002 by Uwe Helmke and Paul Fuhrmann for state space systems. This internal model principle for observers leads naturally to a parametrization of the set of all asymptotic observers for a given controllable system.

This is joint work with Jan C. Willems.

Quaternionic Cayley transforms revisited

Florian Vasilescu

It is well known that the numerical Cayley transform can be extended to not necessarily bounded symmetric operators in Hilbert spaces, by a classical construction due to von Neumann.
The operator Cayley transform does not seem to have useful properties when applied to operators which are no longer symmetric. For this reason, in order to find a similar formula for normal or formally normal operators, in particular for complex numbers, one is leaded to consider a quaternionic framework. An attempt to extend this transform using the context of quaternions has been already made in the past by the author of this text. Recently, we have replaced the previous definition by an equivalent one, whose simplified form allows us to get the properties of the quaternionic Cayley transform from the usual Cayley transform for symmetric operators. Moreover, the new construction does not require densely defined operators, which might be useful for potential applications. As examples, we exhibit some differential operators with 2×2 matrix coefficients.

Joint system realizations, rational solutions of the Schlesinger system and related tau functions

Dan Volok

Origins of the system realization theory may be found in many different fields: for instance, scattering problems in physics, synthesis problems in electrical networks and control problems in linear dynamical systems. For many mathematicians the story begins with the theory of operator colligations and their characteristic functions which was introduced by M. Livšic. Investigations of M. Livšic were continued by L. Sakhnovich, who considered, in particular, the spectral factorization and classical interpolation of a rational matrix function, when both the function and its inverse are given as transfer functions of linear systems, and discovered the key role of matrix equations of the form

$$AS - SB = C$$

(a.k.a. the Sylvester-Lyapunov equations) in these problems. Later system realizations of rational matrix functions and their inverses were given a complete treatment in a series of works by J. Ball, I. Gohberg, M. Kaashoek, L. Rodman *et. al.* on reconstruction of rational matrix functions from the local data - zeros and poles.
It turns out that the above mentioned results are very useful in the study of holomorphic families of rational matrix functions parameterized by the position of zeros and poles, which possess various iso- (from $\bar{\iota}\sigma o\varsigma$ - equal - in Old Greek) properties. In particular, one can construct explicitly rational solutions of the Schlesinger system of partial differential equations. For these solutions the determinant of the matrix S, which satisfies the appropriate Sylvester-Lyapunov equation, coincides with the tau function introduced by M. Sato, T. Miwa and M. Jimbo. The theory also allows to construct explicit counterexamples to the theorems of L. Schlesinger and T. Miwa - that is, isomonodromic families which are *not* solutions of the Schlesinger system and are *not* meromorphic with respect to the parameter.

This is a joint work with V. Katsnelson.

Exact controllability of Schrödinger type systems

George Weiss

(Joint work with M. Tucsnak).

Abstract

We show that if a well-posed system is described by the second order (uncontrolled) equation $\ddot{w} = -A_0 w$ and either $y = C_1 w$ or $y = C_0 \dot{w}$ (y being the output signal) and if this system is exactly observable, then this property is inherited by the system described by the first order equation $\dot{z} = iA_0 z$, with either $y = C_1 z$ or $y = C_0 z$. Such results can be used to prove the exact observability of systems governed by the Schrödinger equation, using results available for systems governed by the wave equation.

Keywords

Second order system, Schrödinger equation, exact observability.

Let H be a Hilbert space, $A_0 : \mathcal{D}(A_0) \to H$ is strictly positive and for all $\alpha > 0$, $H_\alpha = \mathcal{D}(A_0^\alpha)$ with the usual norm. Define $X = H_{\frac{1}{2}} \times H$, which is a Hilbert space with the product norm and $\mathcal{D}(A) = H_1 \times H_{\frac{1}{2}}$. Define $A : \mathcal{D}(A) \to X$ by

$$A = \begin{bmatrix} 0 & I \\ -A_0 & 0 \end{bmatrix}, \quad \text{i.e.,} \quad A \begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} g \\ -A_0 f \end{bmatrix}. \tag{0.3}$$

It is easy to see that A is skew-adjoint. X_1 stands for $\mathcal{D}(A)$ endowed with the graph norm. Our first result concerns the admissibility for observations acting on the first component of the state of the system: this admissibility is inherited by a certain Schrödinger type system.

Proposition 1. Let Y be a Hilbert space, let $C_1 \in \mathcal{L}(H_1, Y)$ and define $C \in \mathcal{L}(X_1, Y)$ by

$$C = [C_1 \ 0]. \tag{0.4}$$

Assume that C is an admissible observation for the unitary group $\mathbb{T}$ generated by A. Let $\mathbb{S}$ be the unitary group generated by iA_0 on $H_{\frac{1}{2}}$. Then C_1 is an admissible observation operator for $\mathbb{S}$.

When we say that (A, C) is exactly observable in time τ, then it is understood that C is an admissible observation operator for the semigroup generated by A.

Theorem 0.1. *With the assumptions in Proposition* 1*, assume that the pair* (A, C) *is exactly observable in some positive time. Then the pair* (iA_0, C_1)*, with the state space* $H_{\frac{1}{2}}$*, is exactly observable in any positive time.*

Now we consider systems where the observation acts on the second component of the state, deriving similar results. We start again with admissibility.

Proposition 2. Let Y be a Hilbert space, let $C_0 \in \mathcal{L}(H_{\frac{1}{2}}, Y)$ and define $C \in \mathcal{L}(X_1, Y)$ by

$$C = [0 \; C_0] \; . \tag{0.5}$$

Assume that C is an admissible observation for the unitary group $\mathbb{T}$ generated by A. Let $\mathbb{S}$ be the unitary group generated by iA_0 on H. Then C_0 is an admissible observation operator for $\mathbb{S}$.

Now comes the corresponding controllability result:

Theorem 0.2. *With the assumptions in Proposition* 2*, assume that the pair* (A, C) *is exactly observable in some positive time. Then the pair* (iA_0, C_0)*, with state space* H*, is exactly observable in any positive time.*

We mention that under a certain assumption on the spectrum of A_0, the converses of the above theorems are also true. For the proofs and for other details (examples) we refer to Chapter 5 of our book [1].

References

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   `http://www.ee.ic.ac.uk/gweiss/personal/index.html`.

Higher rank numerical range and complex valued inner products

Hugo J. Woerdeman

The rank k numerical range of a Hilbert space operator A was introduced by Choi, Kribs and Zyczkowski, motivated by the theory of quantum error correction. It is defined as the set of those complex numbers λ for which there exists an orthogonal projection P of rank k so that $PAP = \lambda P$. The rank 1 numerical range is the classical numerical range which by the Toeplitz-Hausdorff theorem is well known to be convex. By using the theory of Riccati equations the rank k numerical range was shown to be convex as well (see [1] and [3]). For the case of normal matrices the convexity of the higher rank numerical range reduces to a question about null vectors in a complex valued inner product [2], and as such the convexity result gives new results for complex valued inner products for which direct proofs are still elusive.
Part of this talk is based on joint work with Anatolii Grinsphan.

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Fourier transform of "simple" functions,

Yosef Yomdin

The rate of Fourier approximation of a given function is determined by its regularity. For functions with singularities, even very simple, like the Heaviside step function, the convergence of the Fourier series is slow, and their reconstruction from the truncated Fourier data involves systematic errors ("Gibbs effect"). It was recently discovered in the work of D. Donoho, E. Candes, V. Temlyakov, R. de Vore, and others that an accurate reconstruction from the sparse measurements data (in particular, from the truncated Fourier data) is possible not only for regular functions, but rather for "compressible" ones - those possessing a sparse representation in a certain basis. In many important applications (like Image Processing) the linear representation of the data in a certain fixed basis may be not the most natural starting point. Instead, we can approximate the data with geometric models, explicitly incorporating nonlinear geometric elements like edges, ridges, etc. Accordingly, instead of "linear sparseness" we use another notion of "simplicity", based on the rate of the best approximation of a given function by semi-algebraic functions of a prescribed degree. The main subject of the present talk is that such "simple" functions can be accurately reconstructed (by a non-linear inversion) from their truncated Fourier or Moment data.

Derivatives of Meromorphic Functions with Multiple Zeros

Lawrence Zalcman

Recently [1], we proved the following

Theorem. Let f be a transcendental meromorphic function on $\mathbb{C}$, all but at most finitely many of whose zeros are multiple. Then f' takes on every nonzero complex value infinitely often.

In this talk, we describe the background of this result, explain its connection with Hayman's Alternative and Bloch's Principle, and discuss generalizations and related open problems.

References

[1] Shahar Nevo, Xuecheng Pang, and Lawrence Zalcman, *Quasinormality and meromorphic functions with multiple zeros*, Journal d'Analyse Mathématique **101** (2007), 1-23.