OPERATOR THEORY, SYSTEM THEORY AND SCATTERING THEORY: MULTIDIMENSIONAL GENERALIZATIONS AND RELATED TOPICS, 2005

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ABSTRACTS

Nevanlinna-Pick tangential interpolation problems for Schur-Agler-class functions on general domains in $\mathbb{C}^n$

Joe Ball

We consider a bitangential interpolation problem for operator-valued functions defined on domains in $\mathbb{C}^n$ of a certain general class (including as particular cases, Cartan domains of types I, II and III) which satisfy a type of von Neumann inequality associated with the domain (a "Schur-Agler class"). The formulation of the interpolation conditions involves a functional calculus with operator-tuple argument which makes use of the explicit expression of the Taylor functional calculus for a commuting tuple of operators via the Bochner-Martinelli kernel as worked out by Vasilescu. The compact formulation of the interpolation conditions includes prescription of various combinations of functional values and of higher order partial derivatives along left or right directions at a prescribed subset of the domain as particular examples. Using realization results for such a Schur-Agler class in terms of a unitary colligation and the defining polynomial for the domain, necessary and sufficient conditions for the problem to have a solution can be given. If some additional interpolation constraints involving a particular choice of auxiliary function in a kernel decomposition for the solution are specified, then a linear fractional parametrization for the set of all solutions can also be constructed from the interpolation data set. We also discuss extensions of these results to any of the noncommutative Schur-Agler classes introduced by Ball-Groenewald-Malakorn. This talk discusses joint work with Vladimir Bolotnikov of the College of William & Mary, and is also closely related to recent work of Ambrozie, Eschmeier and Timotin.

Low complexity inversion of hierarchical block-band systems

Patrick Dewilde

Systems of linear equations derived from finite element or finite difference partial differential equations exhibit a hierarchical block-band structure under lexicographical ordering. A simple banded matrix (the band need not be regular in shape) is of quasi-separable type and can be inverted by an algorithm exploiting the structure with an algorithmic complexity essentially determined by the size of the band and linear in the size of the matrix. If the matrix is hierarchical block-band, i.e. it consists at the highest level of a band whose elements are again hierarchical banded matrices, then the property that the inversion scheme is essentially determined by the number of non-zero elements in the matrix gets lost, and there is no exact algorithm known to perform low complexity inversion, linear not only in the main dimension, but also linear in the hierarchical sub- dimensions. However, approximate algorithms exploiting the hierarchical quasi-separability are possible. We present some alternatives and discuss their approximative properties.

The generalized Schur transform and related topics

Aad Dijksma

The lecture is based on joint work with D. Alpay (Beer Sheva), T. Azizov (Voronezh), H. Langer (Vienna), and G. Wanjala (Mbarara). The generalized Schur transformation transforms a generalized Schur function $s(z)$, which is not a unimodular constant, into a generalized Schur function $\widehat{s}(z)$. The transformation can be motivated by various analytical and geometrical consequences. In the lecture we concentrate on augmented Schur parameters and the relation between the reproducing kernel Pontryagin spaces $\mathcal{P}(s)$ and $\mathcal{P}(\widehat{s})$ associated with $s(z)$ and $\widehat{s}(z)$. We will also show, and this is the third topic, how the reproducing kernel method leads to the generalized Schur transform for generalized Nevanlinna functions.

Carathéodory-Féjer interpolation in the ball, integral representations, and the moment problem

C. Dubi

We use reproducing kernel Hilbert space methods to solve a Carathéodory–Fejér type interpolation problem with mixed derivatives (corresponding to a so–called lower inclusive set) in the class of Schur multipliers. Related moment problems – corresponding to certain integral representation on the ball– will also be discussed. This is joint work with D. Alpay.

Weyl limit balls for strongly regular canonical systems

Harry Dym

In this talk I will describe the class of strongly regular J-inner matrix valued functions that was introduced and applied to the study of canonical integral and differential systems in a series of joint papers with D. Z. Arov. The main focus will be on a description of Weyl limit balls that emerges from the analysis. The talk is based on joint work with D. Z. Arov.

Carathéodory interpolation on the non-commutative polydisk

Dmitry S. Kalyuzhnyĭ-Verbovetzkiĭ

The Carathéodory interpolation problem in the N-variable non-commutative Herglotz–Agler class and the Carathéodory–Fejér interpolation problem in the N-variable non-commutative Schur–Agler class are posed. It is shown that the Carathéodory (resp., Carathéodory–Fejér) problem has a solution if and only if the non-commutative polynomial with given operator coefficients (the data of the problem indexed by an admissible set Λ) takes operator values with positive semidefinite real part (resp., contractive operator values) on N-tuples of Λ-jointly nilpotent contractive $n \times n$ matrices, for all $n \in \mathbb{N}$.

Deformations of Fuchsian Systems of Linear Differential Equations and the Schlesinger System

Victor Katsnelson

We consider holomorphic deformations of Fuchsian systems parameterized by the pole loci. It is well known that, in the case when the residue matrices are non-resonant, such a deformation is isomonodromic if and only if the residue matrices satisfy the Schlesinger system. Without the non-resonance condition this result fails: there exist non-Schlesinger isomonodromic deformations.

In the present talk we shall introduce the class of the so-called isoprincipal deformations of Fuchsian systems. Every isoprincipal deformation is also isomonodromic. In general, the class of the isomonodromic deformations is much richer than the class of the isoprincipal ones, but in the non-resonant case these classes coincide. A theorem will be formulated that a deformation is isoprincipal if and only if the residue matrices satisfy the Schlesinger system. The theorem holds in the general case, without any assumptions on the spectra of the residue matrices of the deformation. An explicit example illustrating isomonodromic deformations, which are not isoprincipal and do not possess the Painlevé property, will also be given.

This is a joint work with Dan Volok.

Operator quadratic forms and linear fractional relations

Victor Khatskevich

We consider two notions, those of operator quadratic form (QF)

$$P(X) = A + XB + BX * + XDX*,$$

where A, B, C are fixed linear bounded operators acting from one Hilbert space to another, X is an operator variable, and of operator linear fractional relation (LFR)

$$F(X) = \{X' : A + BX = X'(C + DX)\},$$

where A, B, C, D are fixed operators, and $X, X\prime$ are variables. These notions are closely connected, the notion of QF is in somewhat sense a tool for studying the notion of LFR. We get necessary and sufficient conditions to non-emptiness, convexity and compactness in the weak operator topology of the set

$$M = \{X : P(X) \leq 0\}$$

and we apply these result to the study of the dichotomic behavior of dynamical systems.

Spectral analysis of non-linear operators

Yakov Krasnov

We determine the connections between homogeneous of order m non linear operators in n-dimensional space and agreeable m-ary algebras. We study the influence of the Pierce numbers allocation on these algebras in the same fashion as a spectral analysis of a matrices serves the qualitative properties of the linear operators. As an application we mention with stability theory of an nonlinear (quadratic and cubic) ODE's. On the talk will be explain the main ideas of the method and then discuss related algebraic, geometric and ODE's results and conjectures. In the talk will be given an elementary introduction to m-ary algebras mostly in terms similar to the linear algebra terminology.

Ordinary differential operators with singularities and generalized Nevanlinna functions.

Heinz Langer

We consider some second order differential equations and canonical systems with singularities, for which a Titchmarsh-Weyl function can be defined in such a way that it belongs to some generalized nevanlinna class N_κ.

Stochastic linear–quadratic adaptive control: A conceptual scheme

David Levanony

A conceptual adaptive linear–quadratic (LQ) control scheme is proposed. Its derivation is based on a study of a family of *asymptotic maximum likelihood* (AML) estimators, and their associated limit sets. The geometric properties of such limit sets, lead to the formulation of a time–varying, constrained optimization problem, whose solution is an inherently consistent estimate of the system's unknown parameters. When incorporated within a certainty–equivalence adaptive control scheme, these estimates yield optimal long–run LQ closed–loop performance.

A joint work with Peter E. Caines, McGill University.

On Convex Invertible Sets and Cones of a Simple Structure

Izchak Lewkowicz

For a set of matrices $\mathcal{A}_o$ define the following iterative process,

$$\mathcal{A}_{j+1} = \text{conv}(\mathcal{A}_j\ ,\ \mathcal{A}_j^{-1}), \qquad j = 0,\ 1,\ 2\ \ldots$$

assuming all respective inverses exit. The resulting Convex Invertible Set is $\text{CIS}(\mathcal{A}_o) = \bigcup_{j=1}^{\infty} \mathcal{A}_j$. Similarly, $\mathcal{B}_j = \mathbb{R}_+ \cdot \mathcal{A}_j$ and the resulting Convex Invertible Cone is $\text{CIC}(\mathcal{A}_o) = \bigcup_{j=1}^{\infty} \mathcal{B}_j$.

Convex Invertible Cones appear in the context of maximal sets of matrices sharing the same solution to the Sylvester or the Lyapunov equations. If one fix a pair of solutions, a similar result holds for the Riccati equation. Convex Invertible Sets appear in the context of the Matrix Sign Function algorithm for solving these equations. Both CICs and CISs are good examples of rational functions of non-commuting variables.

For a finite integer N, $\bigcup_{j=1}^{N} \mathcal{A}_j \subseteq \text{CIS}(\mathcal{A}_o)$ (similarly $\bigcup_{j=1}^{N} \mathcal{B}_j \subseteq \text{CIC}(\mathcal{A}_o)$) and in general we do not know whenever equality holds. In this talk we concentrate on cases where equality holds for $N = 1$ or $N = 2$.
This talk is partly based on collaborations with Nir Cohen, Leiba Rodman and Elad Yarkoni.

Overdetermined $2D$ systems invariant in one direction and their transfer functions

Andrey Melnikov

The theory of two dimensional ($2D$) overdetermined time-invariant systems has been extensively developed for the last 20 years; it is closely connected to the theory of commuting operators (see [1], [2]). Our goal will be a generalization of some of these results for time-varying systems, which are invariant in one of the variables (say t_1). Such an invariance allows us to perform a partial separation of variables and to define a transfer function, depending on the corresponding spectral parameter (say λ), which will, additionally, depend on the second variable (say t_2).
We develop a realization theory for transfer functions of (conservative) overdetermined $2D$ systems invariant in one direction. A key feature is that multiplication by $S(\lambda, t_2)$ maps solutions of one ODE with spectral parameter λ (the input ODE) to solutions of another ODE with the same spectral parameter λ (the output ODE).
The theory is interesting by itself, especially since it allows us to use frequency domain analysis in a time varying framework. It has also important connections with completely integrable nonlinear PDEs: the so called Lax equation appears naturally, and the passage from the input to the output ODE with a spectral parameter is analogous to the Bäcklund transformation.

References

[1] J.A. Ball and V. Vinnikov, *Overdetermined Multidimensional Systems: State Space and Frequency Domain Methods*, Mathematical Systems Theory, D. Gilliam and J. Rosenthal, eds., Inst. Math. and its Appl. Volume Series, Vol. 134, Springer-Verlag, New York (2003), 63–120

[2] M.S. Livšic, N.Kravitsky, A.S. Markus and V. Vinnikov, *Theory of Commuting Nonselfadjoint Operators*, Kluwer Acad. Press, 1995.

Contractive and completely contractive homomorphisms of planar algebras

Gadadhar Misra

Given a planar domain Ω, we first consider the algebra $\mathrm{Rat}(\Omega)$ of rational functions with poles off Ω and equipped with the norm $\|r\| = \sup\{|r(z)| : z \in \Omega\}$ for $r \in \mathrm{Rat}(\Omega)$ and then its closure which we denote by $\mathcal{A}$. We investigate which contractive homomorphisms of the algebra $\mathcal{A}$ are necessarily completely contractive. We start with homomorphisms $\rho : \mathcal{A} \to \mathcal{L}(\mathcal{H})$ for which $\dim(\mathcal{A}/\ker\rho) = 2$ and show that such a homomorphism is the direct integral of homomorphisms ρ_T induced by operators on a two dimensional Hilbert space via a suitable functional calculus $\rho_T : f \mapsto f(T),\ f \in \mathcal{A}$. It is well-known that contractive homomorphisms ρ_T, where T is from $\mathbb{C}^2$ to $\mathbb{C}^2$ are necessarily completely contractive. Consequently, they possess a dilation. We construct this dilation explicitly. In view of recent examples discovered by Dritschel and McCullough, we know that not all contractive homomorphisms are completely contractive even when $\mathcal{H}$ is finite-dimensional. We explore the possibility, in a certain special case, of constructing a dilation for contractive homomorphisms ρ_T where T is a finite dimensional operator. We construct an operator space which is naturally associated with the problem.

Jump-start in analytic perturbation theory

Boris Pavlov

For the Friedrichs model, obtained as a one dimensional ϵ-perturbation $\mathcal{P}_\epsilon$ of the orthogonal sum of the momentum $\mathcal{P} = i\frac{\mathrm{d}}{\mathrm{x}}$ and a finite Hermitian matrix: $\mathcal{P} \oplus A \to \mathcal{P}_\epsilon$, the scattering matrix is presented as $S_\epsilon(p) = (I + i\epsilon M(p))^{-1}(I - i\epsilon M(p))$. The rational Nevanlinna-class Krein-Weyl function M is associated with the operator A and has poles at the eigenvalues of A. It is proven that for any eigenvalue α_0 of A there exists an *intermediate* operator $\mathcal{P}_0^\epsilon$, which is constructed as a one dimensional perturbation $\mathcal{P}_\epsilon^0$ of the orthogonal sum $\mathcal{P} \oplus A_0^\epsilon$ of the momentum and an appropriate one –dimensional operator,– a solvable model, which plays the role of an intermediate operator in the scattering problem to the pair $(\mathcal{P}_\epsilon, \mathcal{P})$. The scattering matrix S_ϵ^0 to the pair $(\mathcal{P}_\epsilon, \mathcal{P})$ is an analytic function of ϵ and the total scattering matrix to the pair $(\mathcal{P}_\epsilon, \mathcal{P})$ can be factorized as a product

$$S_\epsilon(p) = S_\epsilon^0 S_0^\epsilon$$

where S_0^ϵ is the scattering matrix to the pair $(\mathcal{P}_\epsilon, \mathcal{P})$. It is represented by a single Blaschke factor with a pole and a zero approaching α_0 when $\epsilon \to 0$. The non–analytic factor S_0^ϵ describes creation of the resonance form the eigenvalue α_0 of the matrix A.

On hyperbolic matrix polynomials

Leiba Rodman

It is proved that the roots of combinations of matrix polynomials with real roots can be recast as eigenvalues of combinations of real symmetric matrices, under certain hypotheses. The proof uses a recent resolution of the Lax conjecture, due to Lewis, Parrilo, Ramana, which in turn is based on the work by Helton, Vinnikov. Several applications and corollaries are presented. The talk is based on joint work with L. Gurvits.

Hypercomplex analysis and reproducing kernel Hilbert spaces

Michael Shapiro

The talk is thought to be a survey on the topic stated in the title. With the term hypercomplex analysis it is referred to generalizations of one-dimensional complex analysis which employ the concept of hypercomplex numbers. In the talk we restrict the presentation to different versions of quaternionic and Clifford analysis which are being developed quite intensely the last decades in many centers. A brief account of those versions will be presented, as well as a review of what has been done in the theory of reproducing kernel Hilbert spaces in hypercomplex setting, from its beginning which happened not too long ago and up to the very recent developments.

Dilations of representations of product systems

Baruch Solel

We discuss dilation results generalizing Sz. -Nagy's dilation theorem (for a single contraction) and Ando's theorem (for a pair of commuting contractions) and show that these results lead to dilations of completely positive maps on von Neumann algebras.

On a family of operators appearing in a model of an irreversible quantum system

Michael Solomyak

In the model suggested by the physicist Smilansky one studies an operator describing the interaction between a quantum graph and a system of K one-dimensional oscillators attached at several different points $o_1, \dots o_K$ in the graph. In the talk we discuss the simplest case when the graph coincides with the real line $\mathbb{R}$. For this case the problem reduces to the study of the operator $\mathbf{A}_{\alpha_1,\dots,\alpha_K}$ in the space $L^2(\mathbb{R}^{K+1})$ generated by the differential expression

$$\mathcal{A}U = -\frac{\partial U}{\partial^2} + \frac{1}{2}\sum_{k=1}^{K} \nu_k^2\big(-\frac{\partial^2 U}{\partial {q_k}^2} + q_k^2 U\big)$$

and the conditions

$$U'_x(o_k+, \mathbf{q}) - U'_x(o_k-, \mathbf{q}) = \alpha_k q_k U(o_k, \mathbf{q}), \qquad k = 1, \dots, K.$$

The real parameter α_k expresses the strength of interaction between the quantum graph and the k-th oscillator. Note that the differential expression $\mathcal{A}$ does not contain any parameter, but the "matching conditions", which define the domain of the operator, depend on the parameters.

One has to realize $\mathbf{A}_{\alpha_1,\dots,\alpha_K}$ as a self-adjoint operator and to investigate its spectral properties. In spite of its seeming simplicity, the problem exhibits many unusual effects, even for $K = 1$. The main analytic tool in the analysis of these effects is theory of Jacobi matrices.
For $K > 1$ the study reduces to the case $K = 1$ with the help of various operator-theoretic tools. In particular, scattering theory allows one to give the complete description of the absolutely continuous spectrum of the operator $\mathbf{A}_{\alpha_1,\dots,\alpha_K}$ for all values of the parameters.
A large part of the result was obtained in co-operation with S.N. Naboko ($K = 1$) and with W.D. Evans ($K > 1$).

When nonselfadjoint Hill operators are spectral operators of scalar type?

Vadim Tkachenko

We derive necessary and sufficient conditions for one-dimensional periodic Schrödinger operators $H = -d^2/dx^2 + V$ to be spectral operators of scalar type. The conditions show the remarkable fact that the property of being a spectral operator is independent of smoothness (or even analyticity) of the potential V. The problem of deciding which periodic Schrödinger operators are spectral operators of scalar type appears to have been open for over 40 years.

This is the joint work with F.Gesztezy, University of Missouri, Columbia, USA

Positive maps on algebras of fractions and applications

Florian Vasilescu

Extension result, expressed in terms of complete boundedness, leading to fairly explicit characterizations of the solvability of power moment problems with unbounded operator data are given. As applications, necessary and sufficient conditions for the existence of selfadjoint and normal extensions for some classes of commuting tuples of unbounded linear operators are obtained.

Schur multipliers and de Branges - Rovnyak spaces: non-stationary multiscale case

Dan Volok

In the classical theory of stationary dissipative systems one considers the Hardy space of the unit disk and the operators of multiplication by Schur functions. In the non-stationary case, the Hardy space, the Schur class, the complex variable z and the complex constants are replaced by the upper-triangular Hilbert - Schmidt operators, the upper-triangular contractive operators, the bilateral shift operator Z and the diagonal operators, respectively. As was shown by D. Alpay, P. Dewilde and H. Dym, a suitably defined point evaluation of the upper-triangular operator at the diagonal "constant" can be used to extend many function-theoretical results from the setting of the unit disk to that of the upper-triangular operators.

In this talk we discuss how similar methods can be applied to the study of linear systems indexed by the nodes of a homogeneous tree of order $q \geq 2$, where now q non-commuting shifts all come into play and the commutation relations between these shifts and the space of "constants" are more involved. This is a joint work with D. Alpay and A. Dijksma.