**A Historical Overview of Visualization and Visualizing in Mathematics Education**

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Almost four decades ago I became intrigued by claims that some people strongly preferred to think about mathematics tasks in non-visual ways, even when the tasks seemed to demand visual ways of thinking. I also discovered, through research, that students who did very well on pencil-and-paper “visualization” tests were often among those who preferred **not** to use visual methods when they attempted mathematical tasks; and, curiously, those who did not do well on standard visualization tests often described themselves as “visual thinkers” and preferred to use visual methods when attempting to solve mathematics tasks. Between 1975 and 1982 I saturated myself in the psychology and mathematics education literatures associated with those themes, and became aware of the pioneering work on the nature of spatial ability and mental imagery of psychologists like Jean Piaget, Bärbel Inhelder, Wayne Zimmerman, Vadim Kruteskii, Allan Paivio, Zenon Pylyshyn, Alan Richardson, Roger Shepard, Stephen Kosslyn, John Eliot, and David Lohman, and of mathematics educators like Ian Macfarland-Smith, Richard Skemp and Alan Bishop.

Alan Bishop, then at Cambridge University, was less interested in the pyschologists’ views on visualization, and more interested in visual imagery, and in the impact that visual emphases in teaching and learning mathematics might have on the ways learners thought about mathematical concepts and relationships. Bishop tended to use the word “visualization” differently from the psychologists—he thought of visualization in terms of forming visual images in relation to mathematical concepts and using these images when attempting to solve mathematics problems. Alan had a large influence on Norma Presmeg, whose work on visualization in mathematics has been centrally important in mathematics education since the mid-1980s.

In the early 1990s, the Mathematical Association of America published an edited volume on “Visualization in Teaching and Learning mathematics” (Zimmermann & Cunningham, 1991). In the editors’ introduction to that work, readers were reminded that David Hilbert had spoken of two tendencies in mathematics, one that sought to crystallize logical relations and the other to develop intuitive understanding, especially through “visual imagination” (p. 2). Ted Eisenberg and Tommy Dreyfus made an important contribution to that volume, in an article entitled “On the reluctance to visualize in mathematics” (Eisenberg & Dreyfus, 1991), and have continued to write articles relating to the theme. Eisenberg and Dreyfus spoke of mathematics students preferring “algorithmic over visual thinking” (p. 25), and discussed data which supported that viewpoint.

In this presentation I will survey meanings given, historically, to the term “visualization” in mathematics education, and discuss an ongoing controversy over whether, in fact, mathematics students do prefer algorithmic thinking over visual thinking. It will be argued that, at the very least, mathematics educators need to tighten their working definition of “visualization.” It will also be argued that on the issue of preference for working mode, the little-known (and poorly understood) work of Stephanus Suwarsono should be invoked.