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Complexity: Exercise 2

Due date: 23/11/99. You may submit your exercise in pairs.

Exercise 1
Classify the following languages to one of the following options:

- $L \in \mathcal{R}$.
- $L \in \mathcal{RE}$ but $L \notin \mathcal{R}$.
- $L \notin \mathcal{RE}$.

You must prove your classification! The following languages contain only legal encodings of Turing Machines.

1. $L_1 \stackrel{\text{def}}{=} \{ \langle M \rangle : |L(M)| > 300 \}$.
2. $L_2 \stackrel{\text{def}}{=} \{ \langle M \rangle :$ There is an input x for which M runs more then 300 steps $\}$.
3. $L_3 \stackrel{\text{def}}{=} \{ \langle M \rangle : L(M) = \emptyset$ and $|\langle M \rangle| \leq 300 \}$.
4. $L_4 \stackrel{\text{def}}{=} \{ (\langle M \rangle, k) : L(M) = \emptyset$ and $|\langle M \rangle| \leq k \}$.
5. $L_5 \stackrel{\text{def}}{=} \left\{ \langle M \rangle : \overline{L(M)} \in \mathcal{RE} \right\}$.
6. $L_6 \stackrel{\text{def}}{=} \{ \langle M \rangle : L(M)$ is finite $\}$.
7. $L_7 \stackrel{\text{def}}{=} \{ \langle M \rangle : L(M)$ is infinite $\}$.

Exercise 2
Prove or disprove the following claims:

1. If $\mathcal{C}$ is a non-trivial sub-class of $\mathcal{RE}$ and $\emptyset \in \mathcal{C}$ then $L_{\mathcal{C}} \in \mathcal{RE}$.
2. If $\mathcal{C}$ is a non-trivial sub-class of $\mathcal{RE}$ and $\emptyset \in \mathcal{C}$ then $L_{\mathcal{C}} \notin \mathcal{RE}$.
3. If $\mathcal{C}$ is a non-trivial sub-class of $\mathcal{RE}$ and $\emptyset \notin \mathcal{C}$ then $L_{\mathcal{C}} \in \mathcal{RE}$.
4. If $\mathcal{C}$ is a non-trivial sub-class of $\mathcal{RE}$ and $\emptyset \notin \mathcal{C}$ then $L_{\mathcal{C}} \notin \mathcal{RE}$.

Exercise 3
Let

$$L_{\mathrm{EMP},\overline{\mathrm{EMP}}} \stackrel{\mathrm{def}}{=} \{\, (\langle M_1 \rangle, \langle M_2 \rangle) : L(M_1) = \emptyset \text{ and } L(M_2) \neq \emptyset \,\}\,.$$

Show that if $L \in \mathcal{RE} \cup \mathrm{co}\mathcal{RE}$ then

$$L \leq L_{\mathrm{EMP},\overline{\mathrm{EMP}}}.$$

Exercise 4
The purpose of this question is to explain why in the definition of space complexity we count the memory cells that the machine scans without writing. Let $L \stackrel{\mathrm{def}}{=} \{\, wcw : w \in \{0,1\}^* \,\}$.

1. Show that $L \in \mathcal{DSPACE}(\log n)$.
2. Show that there exists a non-deterministic Turing Machine that writes only on the leftmost cell of its work tape and accepts $\overline{L}$. Your Turing Machine may scan any number of cells in the work tape without writing on them.