Homework assignment no. 1

1. Let P be a set of n points in the plane. Construct a data structure of size $O(n)$ for answering in/out queries in $O(\log n)$ time. That is, for answering queries of the following form: given a query point q, determine in $O(\log n)$ time whether q lies in the convex hull of P.

2. Let C_1 and C_2 be two convex polygons with n_1 and n_2 vertices, respectively. (Each polygon is given by the sequence of its vertices in clockwise order.) Describe an $O(n)$-time algorithm for computing $C_1 \cap C_2$, where $n = n_1 + n_2$.

3. Prove that the problem of computing the convex hull of a set of n points in the plane has an $\Omega(n \log n)$ lower bound. Hint: Show that a set of n real numbers can be sorted in time $O(n)$, plus the time needed for a single convex hull computation.

4. Let S_1 be a set of n disjoint horizontal segments, and let S_2 be a set of n disjoint vertical segments. Describe an $O(n \log n)$-time algorithm for *counting* the number of intersections in $S_1 \cup S_2$.

5. Let D be a set of n unit disks (i.e., disks of radius 1). Define the *depth* of a point p to be the number of disks in D covering p. Describe an algorithm (based on sweeping) that finds a point of maximum depth. What is the running time of your algorithm?

Submission: December 1, 2015.